CHAPTER ONE
DC MACHINE BASICS

DC machines include dc motors and dc generator in both cases, machine operation is based on two fundamental electromagnetic interactions:

I) A conductor moving in a magnetic field will have an electromotive force induced in it.

ii) A conductor carrying current and lying in a magnetic field have a mechanical force developed on it. This chapter reviews these principles, and explains how dc machines are arranged to make use of them.

1.1 Basic Interactions

Consider a straight conductor laying in a magnetic field as in fig. 1.1 the field is uniform along the length of the conductor and perpendicular to it. Let

b =magnetic flux density in tesla, and

L =active length of the conductor in meters.

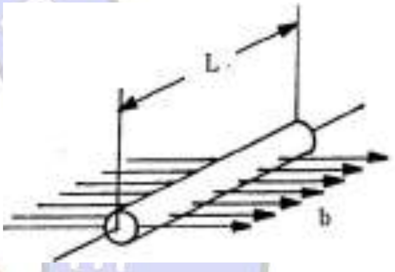


Fig (1.1)

The active length of the conductor is the part which is actually subjected to the field. If, for example, the conductor extends beyond the region of the field, the part that is outside the region is not included in L.

Now, the first electromagnetic interaction occurs if the conductor is moving:

Motion +field ~ induced electromotive force (emf).

The value of the emf is given by:

$$e = ubL\sin(\alpha)\text{volts} \qquad (1.1)$$

Where u =speed of conductor perpendicular ·to its length in meters per second.

and.α =small angle (ie less than 180°) from direction of motion (u) to direction of field b. The direction of the induced emf e is given by the right-hand (RH)rule applied as follows: the fingers of the right hand are extended in the direction or motion in such a way that they can be rotated to the direction of the field through the small angle between them; the extended thumb then gives the direction of the emf in the conductor. Fig. 1.2 shows examples of the application of the RH rule; in fig. 1.2a. e=ubL because sin(α) = 1, but in.fig. 1.2b the emf is less because sin(α)< 1; if the conductor moves horizontally in the figure, there would be no emf induced in it.

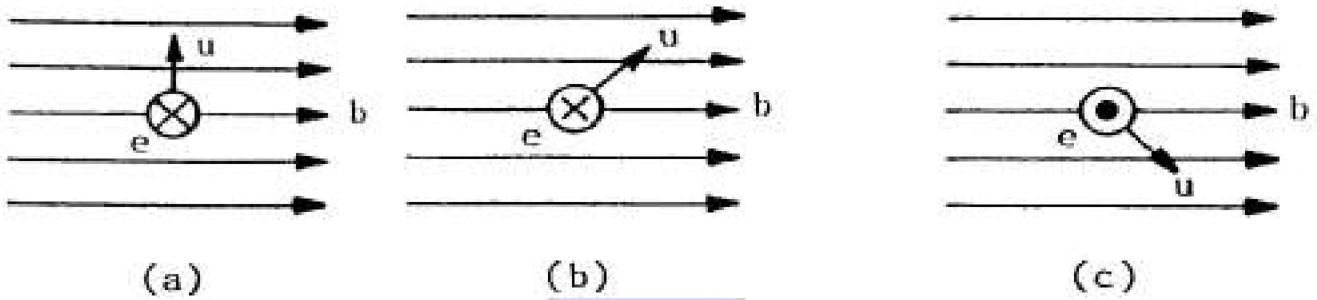


Fig. 1.2 Examples of the application of the right-hand rule to determine thedirection of the emf induced in a conductor moving in a magnetic field

It is important to understand that eqn. 1.1 does not tell us whether or not there is any current in the conductor, nor does it depend on whether the force causing the motion is produced by the conductor itself or by some external means.

The second electromagnetic interaction occurs if the conductor of fig. 1.1carries a current:

current +field ==> mechanical force.

The value of the developed force is given by

f =i bL newtons (1. 2)

Where

i =current flowing in the conductor in amperes.

f acts on the entire active length of the conductor in a direction given by the RH rule as follows : rotate the right-hand fingers from the direction of the current to the direction of the field through the small angle between them; the extended thumb then gives the direction of the developed force.

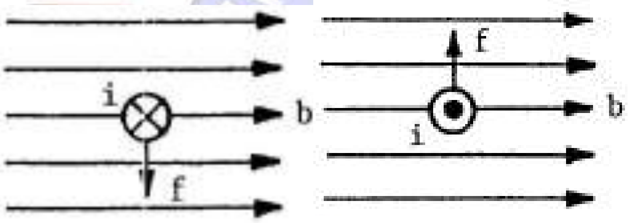


Fig. 1.3 Examples of the application of the RH rule

Fig.1.3shows examples of the application of the RH rule.

It is important to understand that eqn. 1.2 does not tell us whether or not the conductor is moving, nor does it depend on whether the source driving the current is the conductor itself or some external source.

To help you remember the application of the RH rule, note that for both types of interaction, the fingers of the right hand are rotated towards the direction of the magnetic field.

In general, an active conductor in a dc machine will be in motion and carrying current, both at the same time. Therefore, it will have both an induced emf and a developed force. The relative directions of the various quantities involved depend on whether the machine is operating as a generator or as a motor, as we shall see in section 1.7.

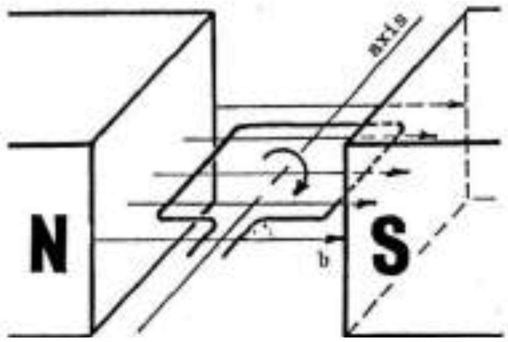


Fig. 1.4 Wire loop in magnetic field.

1.2 Wire Loop

In rotary machines, the active conductors are formed into loops. Fig. 1.4 shows a wire loop placed in a uniform magnetic field. The loop has two active sides marked x and y in fig. 1.5; these are the parts of the loop that are perpendicular to the field and in which the two fundamental interactions occur. The end-connections are necessary to complete the circuit, and the leads are necessary to connect the loop to an external circuit.

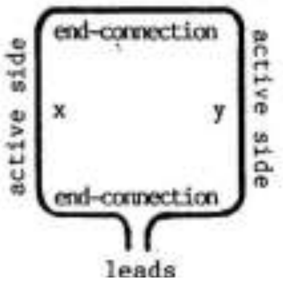


Fig. 1.5 Loop parts.

Let us first see what happens when the loop is rotating about its axis, as shown in cross-section in fig. 1.6a. Here we have motion in a magnetic field, which results in induced emf. Application of the RH rule to sides x and y yields the emf directions shown. The emfs add up around the loop, fig. 1.6b, so that the loop emf e is equal to (e_x+e_y).

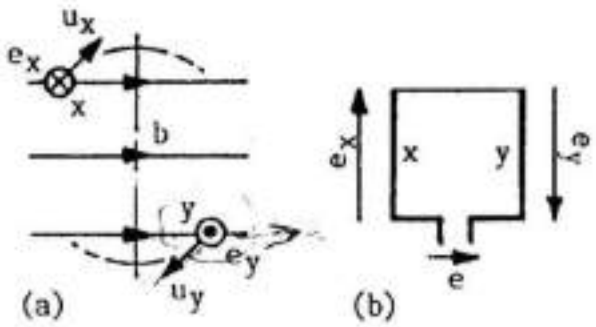


Fig. 1.6 Loop emf due to rotation in uniform field

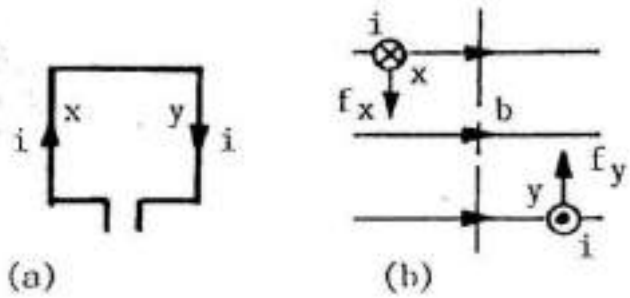


Fig. 1.7 Torque developed by current carrying loop in uniform field.

Next, we see what happens when there is current flowing around the loop, fig. 1.7a.· Here we have conductors carrying current in a magnetic field, which results in forces. Applying the RH rule from side x and y yields forces in the directions shown. The resultant force on the loop is zero because f_x and f_y oppose each other. However, the two forces form a couple, and hence produce a torque about the axis; the developed torque acts on the loop and attempts to rotate it.

1.3 Commutator

The electrical connection between the rotating wire loop and the stationary external circuit requires some form of sliding contact. In principle, this can be achieved using slip-rings and brushes as in fig. 1.8. The slip-rings are made of conducting material, and rotate with the loop; each terminal of the loop is soldered to one of the rings. The brushes are stationary; they are connected to the external circuit. The brushes press against the rings to make sliding contact. Clearly, the terminal emf between brushes e_t is equal to the loop emf e.

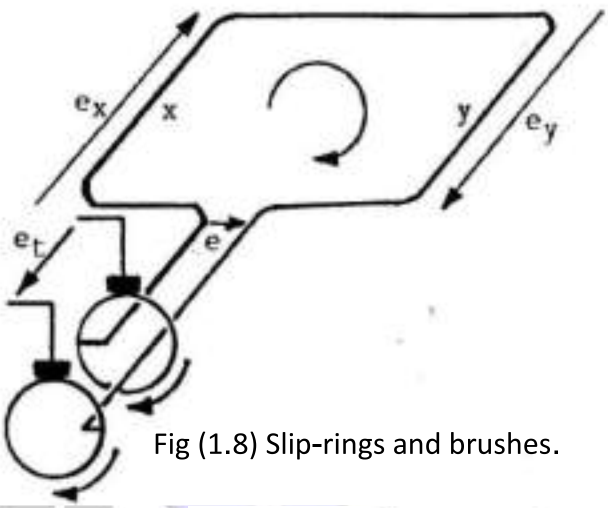


Fig (1.8) Slip-rings and brushes.

But slip-rings are not suitable for dc machines because the loop emf e, and hence the terminal emf e_t, are alternating and not steady as required. The alternating nature of e is demonstrated in fig. 1.9. The emfs in the two sides of the loop, at a number of positions, are shown in fig. 1.9a for rotation in a uniform magnetic field. The figure shows that as the loop passes through the vertical position 3, the emfs in the two sides are reversed, resulting in the alternating wave shown in fig. 1.9b.

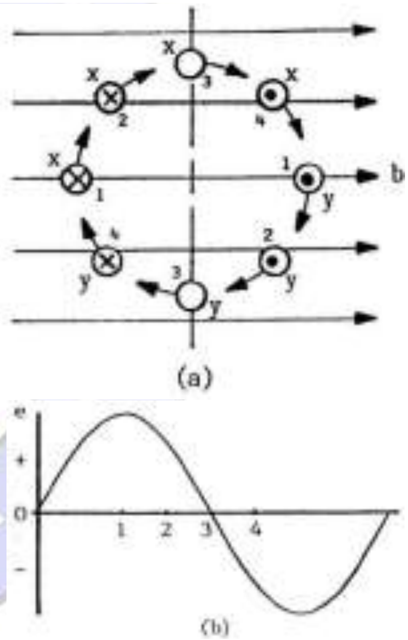


Fig. 1.9 Alternating loop emf due to rotation in uniform field.

The value of the emf at any given position depends on the angle between motion and field, ie α in eqn. 1.1; the emf is zero in the vertical position 3 because, at that position, motion is parallel to the field.

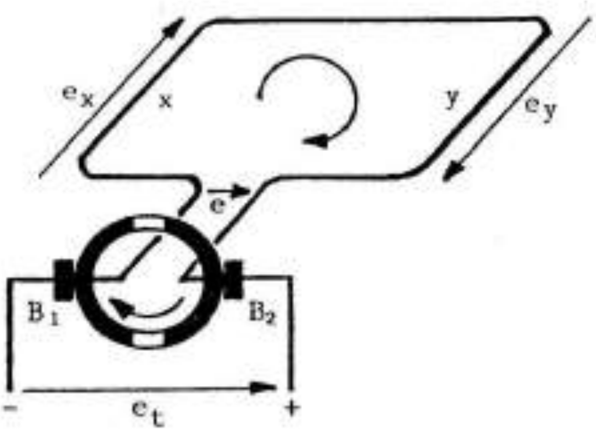


Fig. 1.10 Commutator and

Fig 1.11 Principle of the commutator

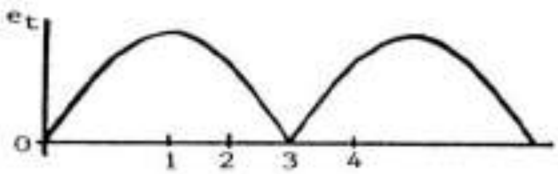


Fig. 1.12 Unidirectional emf obtained between brushes

Thus slip-rings may be used in ac machines, but not in dc machines. DC machines use commutators which not only provide sliding contact, but also rectify the alternating loop emf to a unidirectional emf between brushes. The commutator is basically a conducting ring split into insulated segments; it rotates with the loop, with each terminal of the loop soldered to one of the segments, fig. 1.10. The brushes are stationary, and are connected to the external circuit; they make sliding contact with the commutator segments. The idea of the commutator is that when the loop emf e reverses as the loop passes through the vertical position 3 of fig.1.9, the contact between segments and brushes is also reversed as shown in fig. 1.11. Thus, brush B_1 will always be at low potential, while brush B_2 will always be at high potential. Although the loop emf e is still alternating as in fig. 1.9, the terminal emf between brushes e_t is now unidirectional as in fig. 1.12; that is, e_t is always in the same direction, although its value is not constant.

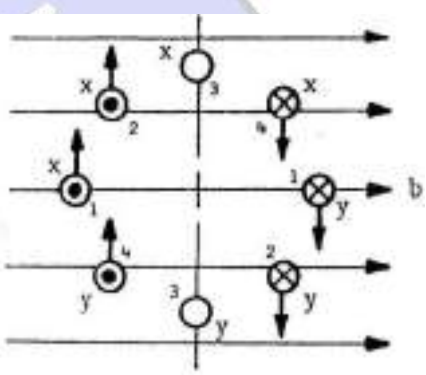


Fig. 1.13 Loop current and developed forces at different positions.

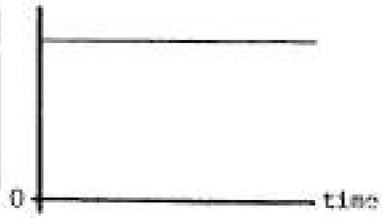


Fig. l.14 Ideal dc conditions

Consider next the effect of the commutator on the developed force and torque. As the machine is a dc machine, the current in the external circuit is always in the same direction. Let us assume that the current from the external circuit goes into brush B_2, flows through commutator and loop, and comes out again through brush B_1. Thus, in positions 1 and 2 of fig.1.13, the current flows into conductor y and out of conductor x; but when the loop passes through the vertical position 3, the contact between segments and brushes is reversed, so that in position 4 the current goes into conductor x and out of conductor y, that is, the loop current is forced to reverse by the commutator and brushes. Fig.1.13 also shows the resulting forces on conductors x and y at the different positions. It is clear that the torque is always in the same direction, clockwise about the loop axis in this case. If the current in the external circuit is truly dc, i.e. not only unidirectional but also constant, the developed torque will have the same wave shape as e_t shown in fig.1.12. If slip-rings are used, the loop current would not reverse at position 3, and the torque would have the wave shape of fig.1.9; that is, the torque would oscillate in both directions, which is useless in a machine.I, e thus conclude that the emf and current inside the loop and the force on its two sides are alternating, but because of the commutator, the emf and current between brushes and the loop torque are unidirectional.

1.4 Armature:

Ideally, the emf and current at the brushes and the torque should be constant with time as in fig.1.14. But we have seen that the emf and torque have the wave shape shown in fig.1.12; although it is unidirectional, it is not constant :it reaches maximum when the loop is in position I of figs.1.9 and 1.13, and goes down to zero when the loop is in position 3. To improve the situation, we can use two loops instead of on1y one, as in fig.1.15. The two loops are displaced from each other by 90^0in space, but are mechanically coupled together to form a single rigid system so that they rotate together.

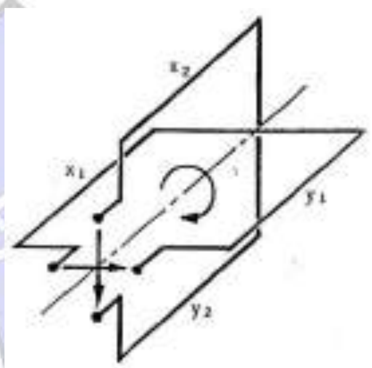


Fig. 1.15 Rigidly coupled loops displaced 90^0

Each loop will give a curve similar to that of fig.1.12, but the two curves will be out of phase :when one loop has maximum emf, the other loop will have zero emf,

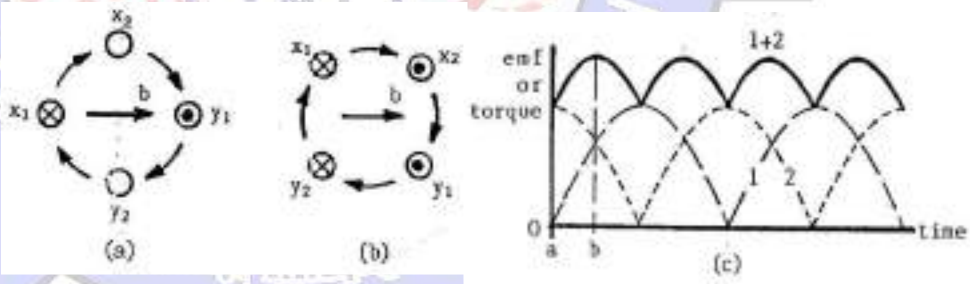


Fig.1.16 Two loops in uniform magnetic field

as can be seen in fig. 1. 16a. now, if the two loops can be connected in series, the total emf will be the sum of the two emfs; also, the two torques add, and their resultant will act on the rigid mechanical system composed of the two loops. Fig. 1.16c shows the curves for the two loops, and also shows their instantaneous sum.

Clearly, the resultant curve is closer to the ideal dc conditions of fig. 1.14than the individual loop curves. The ripple is the instantaneous difference between the curve and its average value; ideally, the ripple should be zero as in fig. 1.14.

The ripple of the resultant curve in fig. l.16b is seen to be smaller than the ripple of the curve in fig. 1.12. The ripple can be made smaller by using more loops with suitable displacements in space; fig. 1.17 shows four loops with 45^0displacements. By using a large number of loops, the ripple can be made very small, practically negligible; however, it cannot be eliminated entirely. The rigid mechanical system composed of all the loops together is called the armature of the dc machine; in most practical machines, the copper loops are mounted on an iron cylinder as we shall see in section 1.5

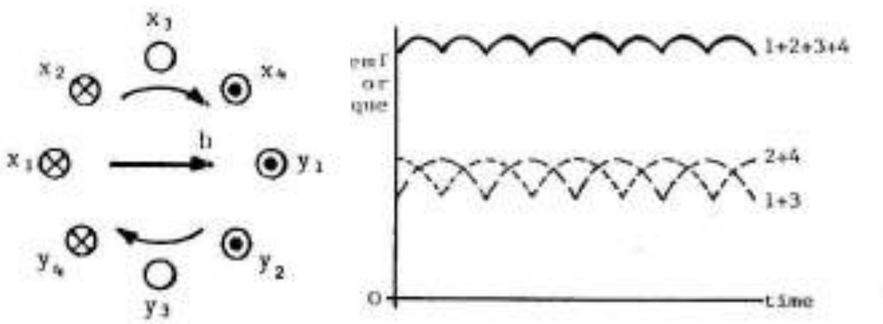


Fig. 1.17 Four loops in uniform magnetic field

In section 1.3, the principle of the commutator was explained for a primitive machine having one loop only. The commutator had two segments, one connected to each terminal of the loop. For practical machines having multiple loops as described here, the commutator has a corresponding number of segments. The necessary number of segments, and the connection of the loops to the segments to ensure that they are in series, will be explained in chapter 3.

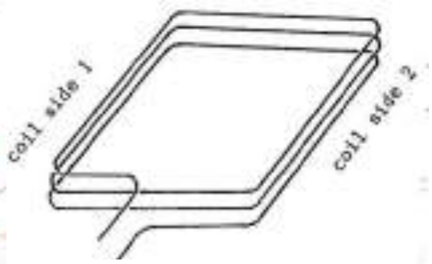


Fig. 1.18 Three-turn coil.

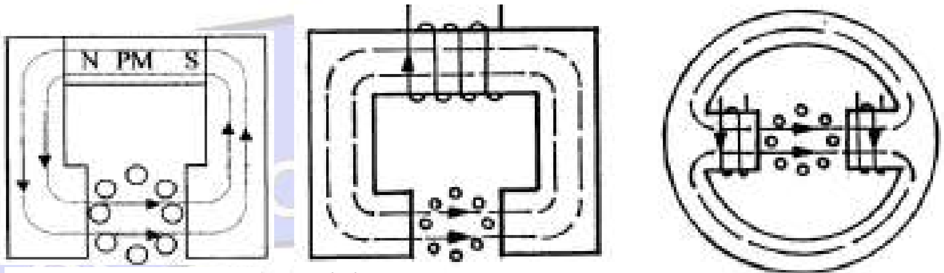
Increasing the number of loops displaced in space not only reduces the ripple, but also increases the emf and torque, as seen in figs. 1.16c and1.17. The emf and torque can also be increased by using multi-turn coils instead of single-turn wire loops.Fig. 1.18 shows a coil having three turns; each turn is in fact a loop, with all loops connected permanently in series so that their emfs add up, and the same current flows through them. Moreover, the loops making up the coil are all located at practically the same position in space, and not displaced from each other as in fig.1.15; in fact, each of the two loops of fig. 1.15 can be replaced by a multi-turn coil. Thus, all the active conductors in each side of the coil see the same flux density, and hence all the turns of the coil have the same emf and the same torque; the loop torques, of course, aid each other, see the same flux density, and hence all the turns of the coil have the same emf and the same torque; the loop torques, of course aid, each other.

For a coil of N turns. There are N active conductors in each coil side as can be seen in fig.1.18. The end-connections at the front and back are necessary to complete the circuit; no useful electromagnetic interactions occur in them. The two Leads of the coil are soldered to commutator segments as was done for the leads of single-turn loops, fig. 1.10.

1.5 Magnetic Field

In the preceding sections we studied the active conductors where the electromagnetic interactions, leading to the production of emf and torque, occur.

Both interactions require the presence of a magnetic field, represented by the magnetic flux density b in eqns. 1 and 2. So far, we have simply assumed that there is a uniform magnetic field in the space around the active conductors or the armature. In this section, we study how the field is produced, and we shall see that it is not truly uniform.



(a) Permenet magnet (b) U- shaped electro-magnet (c) cylindrical electro-magnet

Fig. 1.19 producing the magnetic field.

The field may be obtained by means of a permanent magnet (PM) as in fig.1.19a, or an electromagnet as in fig. 1.19b. In both cases, the iron is shaped to direct the magnetic flux to the region where it is needed, that is, the region where the active conductors of the armature are placed. Fig. 1.19c shows an alternative way of constructing the electromagnet. From the view point of the armature conductors, all three arrangements in fig. 1.19 are the same :they provide a north pole on the left, and a south pole on the right.

Now, the strength of the magnetic field is inversely proportional to the reluctance in its path. In the arrangements of fig.1.19, most of the flux path is in iron, which has a high permeability, and hence low reluctance. But the flux has to pass through the region containing the active conductors; this region is air which presents a high reluctance to the flux. To obtain a strong field, the path in the air must be made as short as possible. By mounting the armature coils on an iron core, as in fig. 1.20b, the air path becomes much shorter than in fig. 1.20a. It can be made even shorter by cutting channels, called slots in the surface of the cylindrical core, fig. 1.20c, and placing the active sides of the armature coils in the slots.

we can seen that the armature is composed of an iron cylinder, with slots in which the coil sides are placed; the end-connections of the coils remain outside the iron cylinder, in front of it and behind it. The whole mechanically-rigid structure is mounted on a shaft through its axis; the torques developed by the coils act on the structure as a whole. Terminals of the coils are soldered to the segments of the commutator, which is also mounted on the same shaft. The clearance between the armature surface and the pole faces is called the air gap. The air gap is necessary to allow the armature to rotate but it is usually made very small so as to obtain the largest possible magnetic flux.

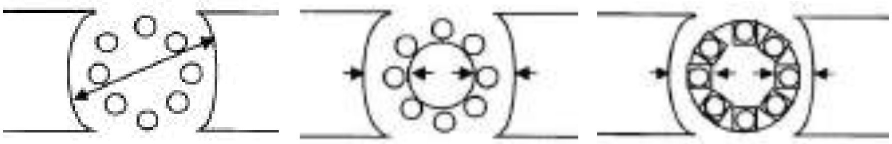


Fig. 1.20 Shortening the air path

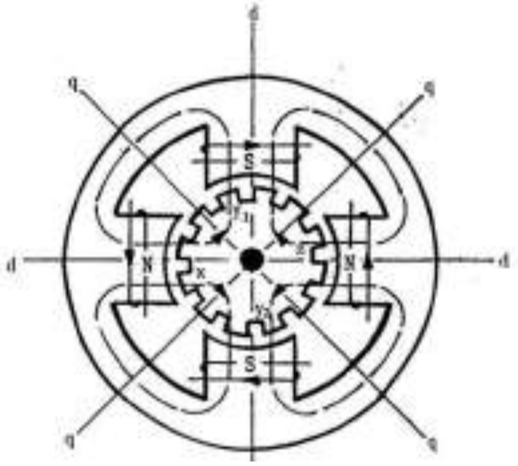


Fig. 1.21Cross-section of a four-pole machine

In figs. 1.19 and 1.20, there are only two poles. One north and the other South; or we can say that the machine has *one* pair of poles. Machines' can have more pole pairs; let

p =number of pole pairs, Then 2p =number of poles,

So that the number of poles is always even :for every north pole, there must be a south pole. Fig. 1.21 shows a machine having four poles, ie. 2p =4, and p =2. Opposite poles follow each other around the armature :every north pole is followed by a south pole, and every south pole is followed by a north pole :note the flux paths in fig. 1.21. The center line of the pole is called the pole axis, or direct axis, or d-axis for shortness; all d-axes are marked with the letter d in the figure. The axis passing through the midpoint between adjacent poles is called the quadrature axis, or q-axis; all q-axes are marked with the letter Q in the figure. Now, let

D= diameter of the armature in meters;

Then the perimeter of the armature is πD meters. This distance faces 2p poles, so to each pole there corresponds a distance of πD/2p meters which is called the pole pitch:

Pole pitch $= \frac{\pi D}{2p}$ meters (1.3)

Clearly, the distance between consecutive d-axes, measured on the armature surface, is a pole pitch; similarly, for the distance between consecutive q-axes. The distance between a d-axis and the adjacent q-axis is half a pole pitch. In fig.1.21, the pole pitch is $\pi D/4$ meters. Sometimes it is useful to measure the pole pitch as a number of slots; in fig. 1.21 there are 12 slots and 4pole. so that the pole pitch is 12/4=3 slots. The pole pitch can also be measured as an angle in degrees or radians :the full circle of the armature is 360 degree (or 2π radians). so that the pole pitch is 360/2p degrees (or$2\pi/2p$ radians). For the 4-pole machine for fig.1.21, the pole pitch is 90degrees (or $\pi/2$ radians). For a 2-pole machine as in figs. l.19 and 1.20, the pole pitch is 180 degrees (or π radians).

Fig. 1.22 shows the emf directions in a 4-pole machine :in each pole pitch corresponding to one pole, i.e. enclosed between two consecutive q-axes, the emfs are in the same direction; the emfs in the next pole pitch are in the opposite direction, and so on. Because of the iron of the armature, the flux tends to be perpendicular to the armature surface, so that the angle between motion and field, α in eqn. 1.1, is 90^0 and $\sin(\alpha) = 1$; thus eqn. 1.1becomes $e = ubL$ for the armature conductors. It is very important to observe that if a coil has one side at the position marked x in figures 1.21 and 1.22, then the second side of that coil must be at position y_1 or y_2 so that the emfs add up around the coil as in fig.1.6b; if the two sides of the coil are placed at x and z, then the emfs oppose each other around the coil, and the coil emf will be zero.

Thus, the distance between the two sides of each coil, which is called the coil span; must be a pole pitch; when one side passes under a north pole, the other side passes under a south pole. Since the pole pitch depends on the number of poles in the machine, eqn. 1.3 then the coil span also depends on the number of poles.

Fig. 1.23 shows the current pattern in the armature conductors and the resulting forces. All the forces are seen to aid each other in developing a torque, CCW in this case. Because the flux is perpendicular to the armature surface, application of the right-hand rule yields forces that are tangential to the surface. Again, it is noted that x and z cannot be the two sides of the same coil; current goes into one side, and comes out through the other, as in x and y_1, or x and y_2.

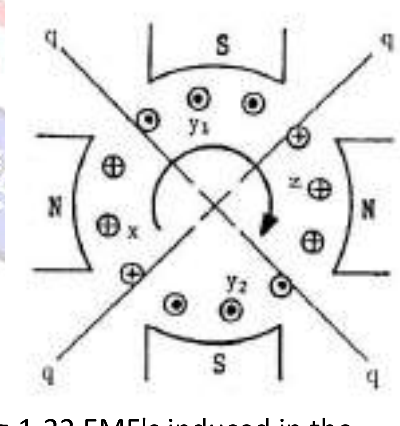


Fig.1.22 EMF's induced in the armature conductors of a 4-pole machine

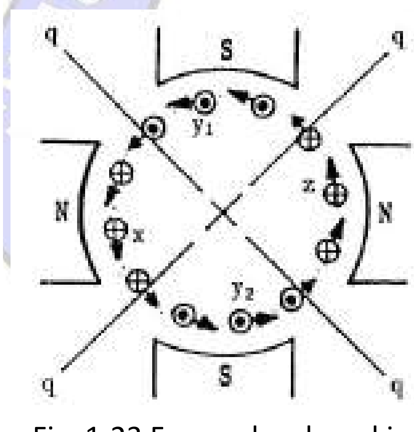


Fig. 1.23 Forces developed in the armature conductors of a 4-pole machine

Conversely, if x and z are assumed to be the two sides of the same coil, then one of the crosses must be changed to a dot, and the forces on the two sides will produce opposing torques; which of course is wrong. Thus, the currents in fig. 1.23 reverse from one pole pitch to the next.

The general path of flux in the machine is shown in fig. 1.21. The actual distribution of field is quite complicated but we shall be mainly interested in the flux density distribution at the surface of the armature because that is where the active conductors are, i.e. where the two main electromagnetic interactions occur; remember that the symbol b in eqns 1.1 and 1.2 represents the magnetic flux density at the location of the conductor. Fig. 1.24 shows a rough sketch of the flux lines in the air gap, ie. at the armature surface.

Note that the machine is shown in fig. 1.24 in developed form, obtained by making an imaginary radial cut in the cross-section shown in fig. 1.21, and straightening the poles and armature; the horizontal length in fig. 1.24 is actually the diameter of the armature.

The curve of the air gap flux density is shown in fig. 1.24. It alternates from one pole pitch to the next because the polarity of the pole's alternates :south followed by north followed by south, etc. Within each pole pitch, the flux density b is high and constant under the pole face because the air gap is the short and has constant length; the absence of pole iron near the q-axis causes the air gap flux density to decrease till it reaches zero at the q-axis; it then reverses as we go into the next pole pitch.

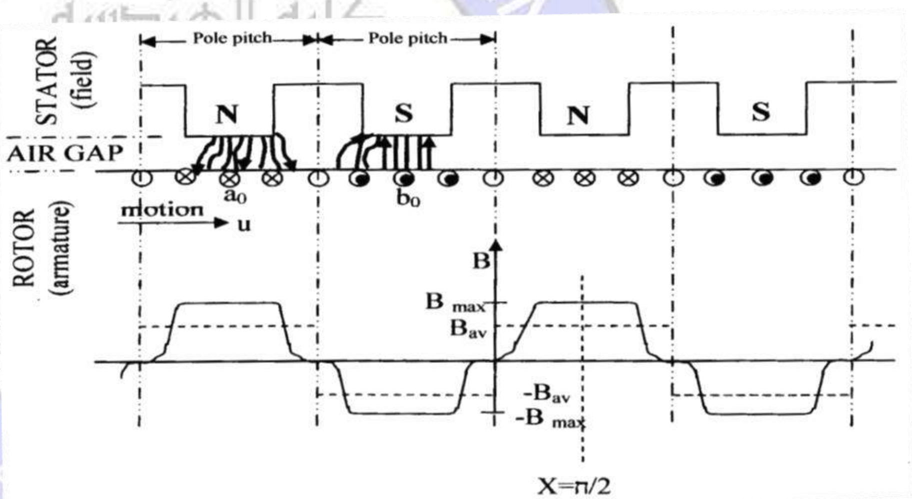


Fig. 1.24 Developed diagram of a 4-pole machine corresponding air gap flux density distribution.

It is clear from fig. 1.24 that the air gap flux density b is periodic :the curve repeats itself every two pole pitches. Thus, electrically, two pole pitches represent 360 degrees (or 2π radians), and we can say that a pole pitch is 180 electrical degrees (or π electrical radians). Care is needed not to confuse the electrical angle described here, and the mechanical angle described all previews page; it should be clear that:

Electrical angle $=p \times$ mechanical angle (1. 4) for example, the full circle around the armature is always 360 mechanical degrees, but it is P x 360 electrical degrees :a 4-pole machine has two electrical cycles around the armature, ie 720 electrical degrees. Moreover.

Pole pitch $=\frac{360}{2p}$mechanical degrees or $=\frac{2\pi}{2p}$mechanical radians (1. 5a)

But, Pole pitch =180 electrica1 degrees or π electrical radians (1.5b)

Note that the pole pitch in electrical degrees (or electrical radians) is constant, and does not depend on the number of poles. Electrically, everything repeats after two poles, so that one pole pitch always represents half a cycle, as expressed by eqn. 1.5b.

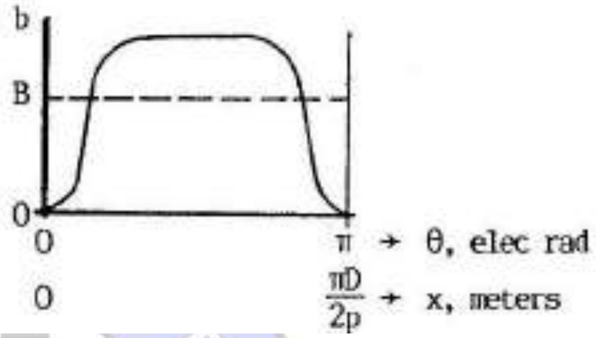


Fig. 1.25 Flux density distribution over one pole pitch.

A typical curve for the air gap flux density is shown in fig. 1.25; the details of the curve may differ a little from machine to machine.

Position along the periphery of the armature be measured as an electrical angle, θ, or as distance in meters, x. Let

B =average flux density in the air gap in tesla.

B is a constant, shown dotted in fig. 1.25. The actual flux density b, on the other hand, is a function of position since it may change from point to point around the periphery of the armature. i.e. b=b(θ), or b=b(x). The average B can be obtained by integrating b over one pole pitch, and dividing by a pole pitch:

$$B=\frac{1}{2\pi}\int_0^{\pi} b d\theta = \frac{1}{\frac{\pi D}{2p}}\int_0^{\frac{\pi D}{2p}} b dx \qquad (1.6)$$

If b is known as a function of position, the above integration can he perform to determine B.

From fig. 1.26, the cylindrical surface area of the armature is πDL. Let:

A_p=area on the armature surface that corresponds to one pole pitch, m^2;

Then

$$A_p = \frac{\pi DL}{2p} \qquad (1.7)$$

Next, let Φ =flux per pole in webers; that is, Φ is the total magnetic flux going out through the face of a north pole and entering the armature, or going from the armature into the face of a south pole. Φ has the same value for all poles. It can be obtained by integrating b over A_p:

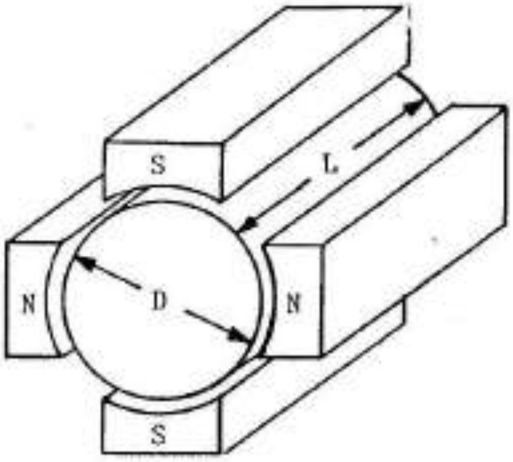


Fig. 1.26 Armature dimensions

$$\Phi = \iint_{A_p} bdA = \int_0^L \int_0^{\frac{\pi D}{2p}} bdxdL \qquad (1.8a)$$

But b is constant along the length of the armature in the direction of L; therefore

$$\Phi = L\int_0^{\frac{\pi D}{2p}} bdx = L\frac{\pi D}{2p}B = A_p x B \qquad (1.8b)$$

Thus, if the average flux density B is known, the flux per pole Φ can be found by multiplying B by the area per pole A_p; conversely, if Φ is known, B maybe found from

$$B = \frac{\Phi}{A_p} \qquad (1.9)$$

This, it will be noted, fits the basic definition of flux density as the flux per unit area.

1.6 Armature Coil Equations

In this section we are going to derive the emf and torque equations for armature coils. The derivations start from eqns. 1.1 and 1.2 which give the instantaneous emf e and instantaneous developed force f_d.

For a given armature conductor, e and f_d vary with time because the conductor sees different values of flux density b as it rotates, fig. 1.24. In fact, e and f_d have the same wave-shape as b in fig. 1.24b since the other factors in eqns. 1.1 and 1.2 are constant; this wave-shape has two kinds of time variation:

(i) It alternates from one pole pitch to the next, and
(ii) Within each pole pitch, it varies between maximum under the pole face and zero at the q-axis.

But we have seen how the commutator rectifiers the alternation, fig. 1.12, and how the use of multiple coils cancels variations about the average, figs. 1.16and 1.17. We are interested in the resulting average commutated values, which are constant, and not in the instantaneous values within conductors, which are time-varying. To distinguish between the two kinds of variables, we shall use capital letters to denote the average commutated values; thus

E =average commutated emf, volts;

F_d = average commutated developed force, newtons;

I= average commutated current, amperes;

T_d= average commutated developed torque, newton meters.

Recall that the average flux density B and the flux per pole Φ are also constant quantities.

1. 6.1 Coil EMF

From eqn. 1.1, the instantaneous emf in an armature conductor is

$$e_{con} = ubL \qquad (1.10)$$

where sin(α) = 1 because the flux is perpendicular to the armature iron. A loop has two active conductors placed a pole pitch apart, so that whatever the flux density seen by one conductor, the flux density seen by the other is equal and opposite, fig. 1.24; thus the two conductor emfs add around the loop, fig. 1.6b, giving

$$e_{loop}=2xe_{cond} = 2ubL \qquad (1.11)$$

A coil having N turns is composed of N loops in series, fig. 1.18, so that

$$e_{coil} = Nxe_{loop} = 2NubL \qquad (1.12)$$

To obtain the average commutated coil emf, we replace the instantaneous flux density b by the average flux density B, so that

$$E_{coil} = 2NuBL \qquad (1.13a)$$

This is the coil emf equation that we require. It can be written in other forms. For example, it is usual to express the speed of rotating bodies in revolutions per second rather than in meters per second. Let

n =rotational speed of the armature, revolutions/second (rps).

Consider a point on the armature surface; the time it takes to complete one revolution is simply (l/n) seconds per revolution. The distance it covers in one revolution is the perimeter of the armature, ie πD. But speed is equal to distance divided by time; therefore

$$u = \frac{\pi D}{\frac{1}{n}} = \pi D n \qquad (1.14)$$

Substituting for u in eqn. 1 .13a, we get

$$E_{coil} = 2\pi DLNnB \qquad (l.13b)$$

Next, let

ω = angular speed of armature, radians/second.

A point on the armature surface covers 2π radians when it completes one revolution in (l/n) seconds; thus

$$\omega = \frac{2\pi}{\frac{1}{n}} = 2\pi n \qquad (1.15)$$

Substituting for n in terms of ω, eqn. 1.13b becomes

$$E_{coil} = DLN\omega B \qquad (1.13c)$$

finally, we can substitute for the average flux density B in terms of the flux

per pole Φ using eqn. 1. 9 in this case, eqn. 1.13b becomes

$$E_{coil} = 2\pi DLNn\frac{\Phi}{A_p} = 2\pi DLNn\frac{\Phi}{\frac{\pi DL}{2p}} = 4pNn\Phi = \frac{2p}{\pi}N\omega\Phi \qquad (1.\ 13d)$$

Eqns. 1.13a. 1.13b. 1.13c. and 1.13d are all expressions for the same quantity, the average commutated coil emf (E_{coil}). In each form, there is a factor representing speed, and a factor representing the magnetic field, multiplied by some constant; this reflects the fact that, fundamentally, the emf is induced by the interaction between motion and field.

It is instructive to show how the expression for E_{coil} can be derived in an entirely different way. Faraday's law states that time-varying magnetic flux induces emf in a coil enclosing it.

$$e_{coil} = N\frac{d\Phi}{dt} \qquad (1.16)$$

where Φ is the flux linking the coil, that is, the total flux passing through the space enclosed by the N turns of the coil. Note that this is not the same as Φ which represents the flux per pole in the machine. Eqn. 1.16 gives the instantaneous emf; the average emf over an interval Δt is obtained by simplifying eqn. 1.16 to

$$E_{coil} = N\frac{d\Phi}{\Delta t} = N\frac{\Phi_2-\Phi_1}{t_2-t_1} \qquad (1.17)$$

Where Φ_1 is the flux linking the coil at the instant t_1 and Φ_2 is the flux linking it at t_2. Let us take t_1 as the instant at which the coil sides are at consecutive q-axes enclosing a north pole as in fig. 1.27a; the flux linking the coil is the flux per pole Φ directed downward, so that Φ_1=-Φ.

Next, we take t_2 as the instant at which the coil has moved so that it encloses the following south pole, fig. 1.27b; the flux linking the coil is still Φ, but it is directed upward so that Φ_2 = Φ Eqn. 1.17 now becomes:

$$E_{coil} = N\frac{\Phi-(-\Phi)}{t_2-t_1} = N\frac{2\Phi}{\Delta t} \qquad (1.18)$$

Now, Δt =t_2-t_1 is the time it has taken the coil to move one pole pitch. As we have seen, the time to complete one revolution is (l/n) seconds. But one revolution covers 2p poles, so that the time to move one pole pitch is:

$$\Delta t = \frac{1/n}{2p} = \frac{1}{2pn} \qquad (1.19)$$

Substituting this value for Δt back into eqn. 1. 18, we get

$$E_{coil} = N\frac{2\Phi}{(1/2pn)} = 4pNn\Phi$$

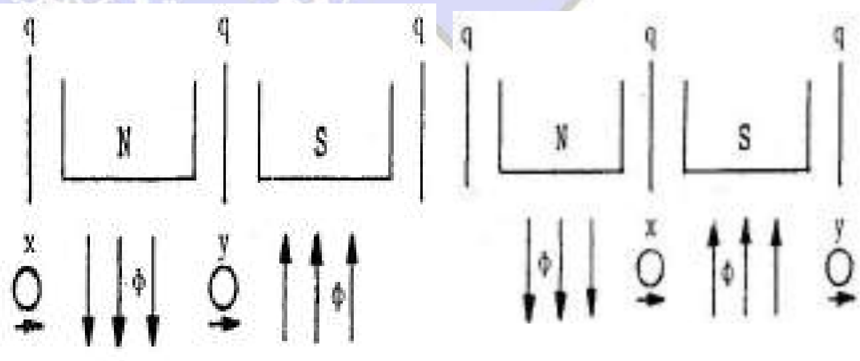
Which is the same as eqn. 1.13d.

1.6.2 Coil torque

From eqn. l.2, the instantaneous force developed in an armature conductor is

$f_{cond} = i_{cond}\, bL$ (1. 20)

where i_{cond} is the current flowing in the conductor. Because the



(a) coil position at instant t_1 (b)coil position at instant t_2

Fig. 1.27 Motion of an armature coil through one pole pitch.

flux is perpendicular to the armature iron, this force is tangential to the armature surface as shown in fig. 1.23. Therefore, this force produces a torque about the axis of the armature

$\tau_{cond} = f_{cond} \text{ x } D/2 = \frac{1}{2} i_{cond} bLD$ (1. 21)

where D/2 is the torque arm. The two active conductors in a loop carry equal and opposite currents, fig. 1.7, and see equal and opposite values of flux density, fig. 1.24, so that them torques are equal and in the same direction about the armature axis, fig. 1.23; thus

$\tau_{loop} = 2 \text{ x } \tau_{cond} = i_{cond}\, bLD = i_{loop}\, bLD$

Where $i_{loop} = i_{cond}$. A coil of N turns is composed of N loops in series, so that $i_{coil} = i_{loop}$ and the total torque developed by the coil is:

$\tau_{coil} = N \text{ x } \tau_{loop} = NDLi_{coil}\, b$ (1. 23)

τ is the instantaneous torque. The average commutated torque is obtained by replacing the instantaneous i and b by I and B:

$T_{coil} = NDLI_{coil}B$ (1.24a)

This is the coil torque equation that we require. It can be written ina different form by substituting for B in terms of Φ using eqn. 1.9:

$T_{coil} = NDLI_{coil}\frac{\Phi}{A_p} = NDLI_{coil}\frac{\Phi}{\frac{\pi DL}{2p}} = \frac{2p}{\pi} NI_{coil}\Phi$ (1.24b)

Eqns. 1.24a and 1.24b express the same quantity, the average commutated coil torque T_{coil}In both forms, current appears as a factor multiplied by a factor representing the magnetic field; this is to be expected because, fundamentally, the torque is produced by the interaction between current and field.

1.6.3 Coil Resistance

The resistance of a conductor of length ℓ meters and cross-sectional area s $m^2$is given by

$R = \frac{\rho L}{s}\ \Omega$ (1. 25)

Where ρ is the resistivity of the material from which the conductor is made.

Machine coils are usually made from copper, for which

$\rho = 1.72 * 10^{-8}\ \Omega m = 1.72\ \mu\Omega cm$ at 20^0 C

The resistivity, and hence resistance, increase with temperature according to the ratio

$$\frac{\rho_2}{\rho_1} = \frac{R_2}{R_1} = \frac{T_2 - T_0}{T_1 - T_0} \qquad (1.26)$$

Where ρ_1 and R_1 are at a temperature T_1 , and ρ_2 and R_2 are at a temperature T_2.T_o is a constant of the material; for copper it is

T_0=-234.5^{0}C

Thus, if the resistivity is known at some temperature T_1, it can be found at a different temperature T_2 using eqn. 1.26; similarly, for the resistance ℓ in eqn. 1.25 represents the total length of the conductor. For a wire loop as shown in fig. 1.5,

$$\ell_{loop} = 2L + \ell_{end} \qquad (1.\ 27)$$

Where ℓ_{end} represents the length of the end-connections at the back and front. Note that the entire length of the loop contributes to its resistance, whereas only the active parts contribute to the emf and torque. Thus

$$R_{loop} = \frac{\rho \ell_{loop}}{s} \qquad (1.28)$$

As a coil is made of N loops in series, the coil resistance is given by

$$R_{coil} = N x R_{loop} = N \frac{\rho \ell_{mean}}{s} \qquad (1.29)$$

Where ℓ_{mean} is the mean length of the turns of the coil, since, for a coil of many turns, not all the turns will have exactly the same length.

1.7 Electromechanical Power Conversion

The voltage between the two terminals of an armature loop or coil is equal to the emf induced in the coil so long as there is no current flowing in it. When current does flow in the coil, it causes a voltage drop due to the resistance of the coil, and the terminal voltage will be somewhat different from the emf.

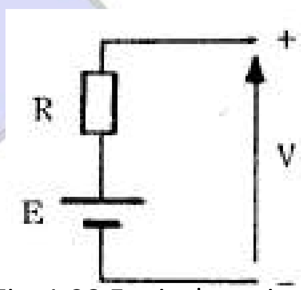


Fig. 1.28 Equivalent circuit of an armature coil

Electrically, then, the coil is represented by a simple equivalent circuit composed of the coil emf in series with the coil resistance, as shown in fig. 1.28. For simplicity, in this section we shall drop the subscript 'coil' on the understanding that all quantities of interest (E, R, V, I, and T) correspond to an armature coil. It is recalled that the emf E is induced in the active sides of the coil due to their rotation in a magnetic field; the resistance R, and hence the voltage drops IR, exist along the entire length of the wire from which the coil is made, including both the active parts as well as the end-connections and leads.

For any circuit element having two terminals, the power is the product of the voltage between the terminals and the current through the element. The power in the emf-element of fig. 1.28 is thus given by

$P_c = E\,I$ (1. 30)

Where E and I are the coil emf and current. If we now substitute for E from eqn. 1.13d, and collect terms, we find

$$P_c = \frac{2p}{\pi} N\omega\Phi I = \omega\left\{\frac{2p}{\pi} NI\Phi\right\}$$

The quantity in brackets is the coil torque as given in eqn. 1.24b; therefore

$P_c = \omega T$ (1.31)

But the product of angular speed and torque is the mechanical power of a rotating system Thus, eqns. 1.30 and 1.31 tell us that the electrical power in the emf-element of the equivalent circuit of the coil is the same as the mechanical power of the rotating coil:

$P_c = EI = \omega T$ (l .32)

These equation summaries the electromechanical power conversion that occurs in the coil by means of the two fundamental electromagnetic interactions of emf and torque production. P_c is called the conversion power; it is equal to EI on the electrical side, and ωT on the mechanical side. It is the power that is changed from mechanical form to electrical form in a generator, and from electrical form to mechanical form in a motor. Let us look at the conversion processes of the two cases in more detail.

1.7.1 Generator Action

A generating coil acts as a source in the electrical circuit :it supplies electrical power to the circuit, fig. 1.29a. The emf-element supplies P_c =EI to the circuit; part of this power is lost as heat in the resistance of the coil itself, I^2R, and the rest goes to the electrical load, VI. Conservation of energy and power requires that the power fed into the circuit must come from somewhere; in the generating armature coil, it comes from the mechanical side as $P_c=\omega T$. Therefore, there must be a mechanical source, or prime mover, providing the mechanical power to rotate the coil.

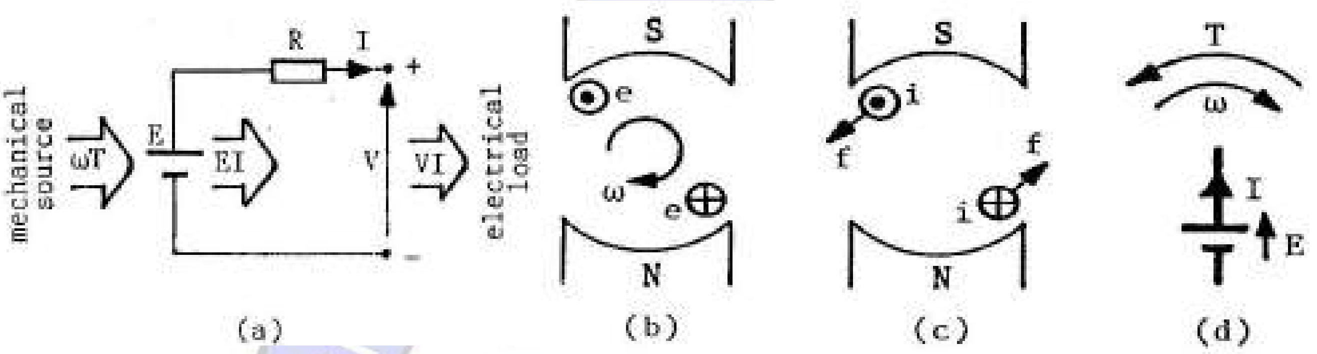


Fig. 1.29 Rotating armature coil in generating mode

Assuming the coil is rotating in the CW direction, application of the RH rule at the instant shown in fig. 1.29b -gives the instantaneous emf directions in the two sides of the coil. Since the coil is generating, it acts as the source in the electrical circuit, and hence drives the current in the same direction as the emf as shown in the equivalent circuit of fig. 1.29a. Thus, the current in the coil sides shown in fig. 1.29c is in the same direction as the emf's shown in fig. 1.29b. But current in a magnetic field produces force, and application of the RH rule gives the force directions shown in fig. 1.29c.

Clearly, the torque is in the CCW direction, that is, the developed torque opposes the rotation. Therefore, there must be an external torque that forces the coil to rotate in the CW direction; the mechanical source provides this torque which acts against the opposition of the torque developed electro-magnetically in the coil.

Fig. 1.29d gives the relative directions for the generating mode. On the electrical side, the emf drives the current into the load so that

E and I are in the same direction;

on the mechanical side, the developed torque opposes rotation so that

T and ω are in opposite directions.

Finally, let us write down the circuit equation for the generating coil. Applying Kirchhoff's voltage law to the equivalent circuit of fig. 1.29a. We have

$$V = E - IR \tag{1.33}$$

That is, the terminal voltage V is less than the induced emf by the amount of voltage drop in the coil resistance IR. Multiplying throughout current by the I, eqn. 1.33 gives

$$VI = EI - I^2R \tag{1. 34}$$

The output power going to the load VI is less than the conversion power $P_c = EI$ by the amount of power lost as heat in the coil itself I^2R. We shall see in later chapters that the process of electromechanical conversion is accompanied by other types of loss, such as friction between moving parts, and hysteresis and eddy currents in iron cores.

1.7.2 Motor Action

A motoring coil acts as a load in the electrical circuit :it receives electrical power from the circuit, fig.1.30a. The electrical source supplies the input power VI to the circuit; part of this

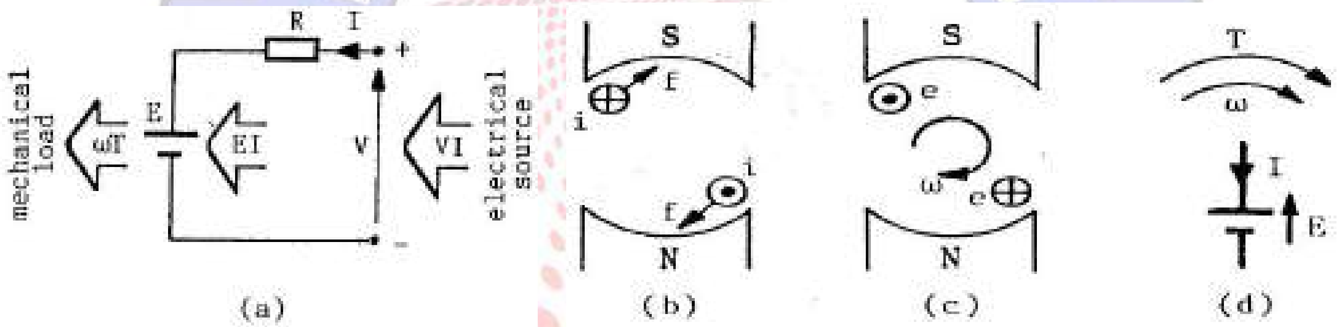


Fig. 1.30 Rotating armature coil in motoring mode

power is lost as heat in the resistance of the coil, $I^2 R$, and the rest goes to the emf-element of the coil as the electric power EI. Conservation of energy and power requires that the power fed into the emf-element of the circuit must go somewhere; in the motoring coil, it goes to the mechanical side as $P_c = \omega T$, where it rotates the mechanical load.

Let us assume that at a certain instant the coil current is in the direction shown in fig. 1.30b; application of the RH rule then gives the force directions in the two sides of the coil, and these produce a CW torque. Since the coil is motoring, it causes the rotation of the shaft and load; that is, rotation is in the same direction as the torque developed electromagnetically, which is CW as shown in fig. I.30c. But motion in a magnetic field induces emf, and application of the RH rule gives the instantaneous emf directions shown in fig. 1.30c. Clearly, the induced emf opposes current.

Therefore, there must be an external voltage applied to the electric circuit that forces the current to flow against the emf; the electrical source provides this voltage which acts against the emf induced electromagnetically in the coil.

Fig. 1.30d gives the relative directions for the motoring mode. On the electric side, the external source forces the current against the induced emf so that

E and I are in opposite directions;

on the mechanical side, the developed torque causes the rotation, so that

T and ω are in the same direction.

The circuit equation for the motoring coil is obtained by applying Kirchhoff's voltage law to the equivalent circuit of fig. 1.30a:

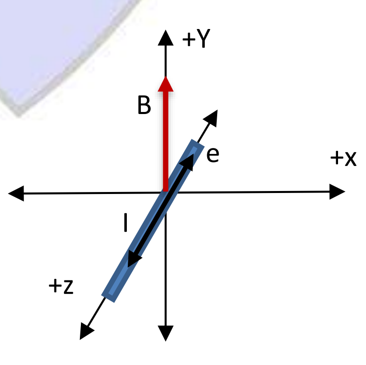
$V = E + IR$ (1.35)

The terminal applied voltage V must be greater than the induced emf E by the amount of voltage drop in the coil resistance IR. Multiplying throughout by the current I, eqn. 1.35 gives

$VI = EI + I^2R$ (1. 36)

The input power from the electric source VI is greater than the conversion power Pc =EI by the amount of power lost as heat in the coil, I^2R.

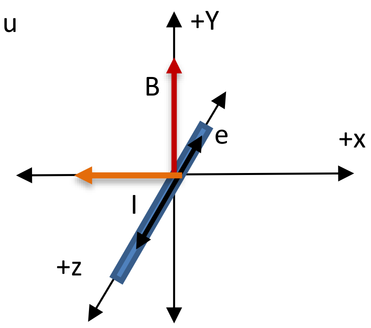
Example1: A straight conductors moving at constant speeds through uniform magnetic fields. Active length 60(z) mM, Flux density 150(+y)m T, Resistance 10 mΩ, Current 0.5(+z) A, Induced emf, 40(-z) mV The indicated directions follow a right-hand Cartesian coordinate system. Find



a) Velocity and direction.
b) What action? and why?
c) Terminal voltage.
d) Developed force and direction.
e) Conversion power.
f) Copper loss.
g) What happen when the direction of flux is reversed?

Solution:

a) $u = \frac{e}{BL} = \frac{40*10^{-3}}{150*10^{-3}*60*10^{-3}} = 4.44$ M/s (-x)



1- By applying the RH rule, the conductor must be moving perpendicular to the direction of the flux B, so the moving of velocity must in the direction of x or z axis.
2- The conductor is in the long z axis so the moving must in x axis only.
3- From 1 and 2 can to get the direction of velocity u moving the fingers from I to B then **get u in the direction of (-x)** also can conclude the direction of emf e is in the (-z).

b) From a can conclude **the action is motor** because of the direction of current is opposite to the direction of emf.
c) For motor $v = e + IR = 40 * 10^{-3} + 0.5 * 10 * 10^{-3} = 45 * 10^{-3} V$
d) $F = i BL = 0.5*150*10^{-3}*60*10^{-3} = 0.0045$ N (-x) in motor action the force is in the direction of velocity (-x).
e) $Pc = EI = 40*10^{-3}*0.5 = 20*10^{-3}$ W
f) Copper loss $= I^2R = 0.5^2*10*10^{-3} = 2.5*10^{-3}$ W
g) When the direction of the flux is reversed the direction of rotation speed is reversed also (+x) using RH rule.

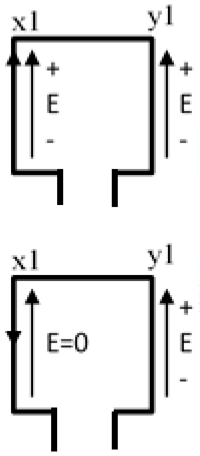
Example2: the coil span should be approximately equal to a pole pitch. Discuss the effect of making the coil span very different from the pole pitch on (a) the coil emf and (b) the coil torque. Use a diagram like that shown in fig. 1.24, and assume first a short-pitched coil spanning half a pole pitch, then a long-pitched coil spanning one and a half pole pitches.

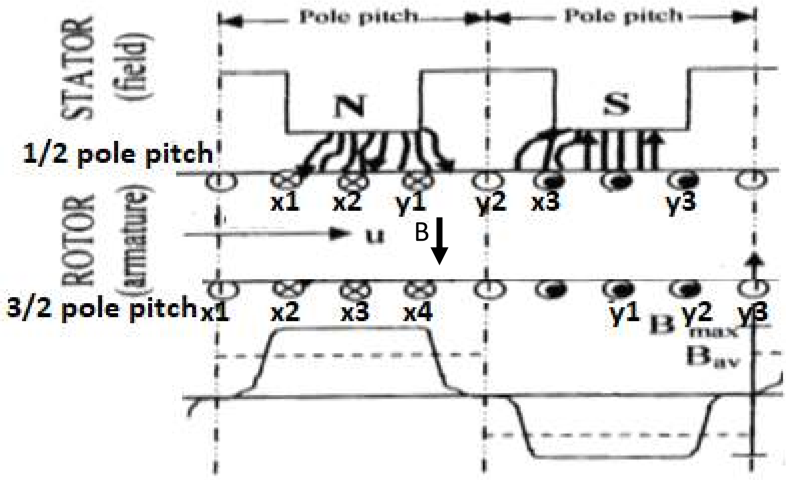
Solution:

a short-pitched coil spanning half a pole pitch:

as see in figure for a short-pitched coil spanning half a pole pitch the emf in the coils x1-y1 that is laying under the same pole will introduce the same voltage but opposite polarity x1 and y1 so the resultant emf introduced is zero $E_{x1} - E_{y1}$, the also the resultant force is zero because no current will passing through the coil, in the other coils x2-y2 the emf will introduced in x2 but in y2 there is no voltage introduced that is mean the resultant emf is ½ of coil emf also the torque introduced is in the part x2 only and zero in part y2 but as a resultant the torque is ½ T_{coil} . a long-pitched coil spanning one and a half pole pitches:

we can see from figure the coil side x1 is lying between poles so no magnetic field will be cutting the conductors but the other side y1 is laying under the pole and will introduced emf so the resultant is ½ emf of the coil and ½ T_{coil}. The coil x2-y2 will introduced emf coil and T_{coil} because of each coil side is under different pole. The coil x3-y3 is the same of coil x1-y1 but in other polarity. The coil x4-y4 is under the pole polarity so the induced emf is zero also the torque.





Exercise

1.1: The table below relates to straight conductors moving at constant speeds through uniform magnetic fields. The indicated directions follow a right-hand Cartesian coordinate system (a screw turning form x to y advances with z). Complete the table giving values, as well as directions where applicable.

	I	ii	iii	Iv	V	vi
Flux density, m T	80(+z)	80(+z)	300(-y)	150(+y)	?	120(+y)
Active length, mM	300(y)	300(y)	150(x)	60(z)	250(x)	200(z)
Resistance, mΩ	5	5	8	10	0.8	2.5
Velocity, m/s	1.6(+x)	1.6(-x)	?	?	?	2(-x)
Current, A	2(-y)	2(-y)	?	0.5(+z)	12(-x)	4(-z)
Induced emf, mV	?	?	90(-x)	40(-z)	-x	?
terminal voltage, mV	?	?	105	?	60	?
Developed force, mN	?	?	?	?	750(+z)	?
Conversion power, mW	?	?	450	?	840	?
Copper loss, mW	?	?	?	?	?	?
Input power. mW	?	?	?	22.5	?	?
Output power, mW	?	?	?	?	?	?
Action	?	?	mot	Mot	Gen	?

1.2 The armature of an 8-pole dc machine has 72 slots, and its diameter is48 cm. Find the pole pitch in (a) meters, (b) mechanical degrees, (c) mechanical radians, (d) electrical degrees, (e) electrical radians, and (f)number of slots. Also find (g) the electrical angle corresponding to a mechanical angle of 35^0, (h) the mechanical angle corresponding to an electrical angle of 35^0, and (i) the distances along the armature surface corresponding to the angles in g and h.

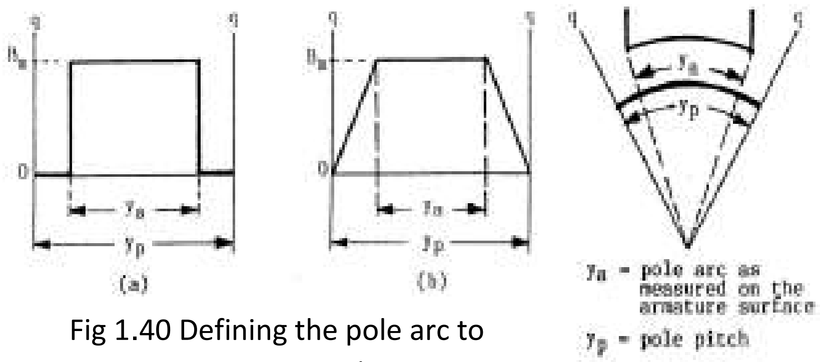


Fig 1.40 Defining the pole arc to pole pitch ratio, $\alpha = Y_a/Y_p$.

1.3 The slot pitch is the distance from the centre of one slot to the centre of the adjacent slot, measured along the surface of the armature. For the armature of question 1.2, find the slot pitch in (a) meters, (b) mechanical degrees, and (c)electrical degrees.

1.4 (a) Assuming that the air gap flux density of fig. 1.25 can be approximated as in fig. 1.40a, shows that the flux per pole and the average air gap flux density are given by:

$$\Phi = \alpha\pi B_m D\ell /2p \quad \& \quad B_{av} = \alpha B_m \qquad ,(\alpha = y_a/y_p)$$

(b)Assuming that the air gap flux density of fig. 1.25 can be approximated as in fig. 1.40b, shown that the flux per pole and the average air gap flux density are given by.

$$\Phi = (\alpha+1)\pi B_m D\ell /4p \quad \& \quad B_{av} = (\alpha+1)B_m/ 2 \qquad , (\alpha = y_a/y_p)$$

1.5 Assuming that the air gap flux density of fig. 1.25 can be approximated by a sine wave, show that the flux per pole and the average air gap flux density are given by:

$$\Phi = B_m D\ell /p \qquad \& \quad B_{av} = (2/\pi) B_m$$

where B_m is the maximum flux density.

1.6 The armature of a 6-pole dc machine is 80 cm long and has a diameter of 50 cm. The maximum air gap flux density is 1.5 T. The pole arc covers70 %of the pole pitch (see fig. 1.40). The armature speed is 500 rpm (revolutions per minute). Assume the air gap flux density is distributed as in fig. 1.40b.

a. Find the flux per pole and the average air gap flux density.

b. Find the emf, developed torque, and conversion power for a wire loop on the armature carrying a current of 9 A.

1.7 The armature of a 4-pole dc machine rotates at 840 rpm. The armature length and diameter are 40 cm and 30 cm respectively. The flux per pole is 65 mWb. Each armature coil has 5 turns, and develops an average torque of 3.0 Nm.

a. Find the coil emf, current, and conversion power.

b. Make a rough estimate of the copper (I^2 R) loss in the coil assuming it is made of 1.0 mm ^{2} copper wire, and the losses during operation raise the temperature to 80 ^{0} c.

1.8 The armature of a small 2-pole dc machine rotates at 1200 rpm. The armature length and diameter are 22 mm and 26 mm respectively. Each armature coil is made of 30 turns, with a resistance of 4 mΩ per turn. The coil terminal voltage and induced emf are 2.0 V and 1.8 V respectively. Find the average air gap flux density and the flux per pole. Also find the coil torque and conversion power.

1.9 A constant dc voltage of 10 V is applied, through brushes and commutator, to a7-turn coil placed on n 4-pole armature of length and diameter 24 cm and 36 cm respectively. The coil resistance is 50 mΩ, and the average flux density in the air gap is 658 mT.

a. finds the average developed torque when the armature rotates at 240 rpm.

b. Find the average developed torque when the speed is (i)decreased by 10%, and (ii)increased by 10.%

c. Find the copper (I^2R) loss in the coi1 for each of the three cases in parts a and b.

d. What is the mode of operation of the coil in each of the cases in parts a and b ?

e. Derive an equation that gives the coil torque as a function of speed; sketch the relationship.

1.10 If the diagrams in figs. 1.6 and 1.7 correspond to the same armature loop, is that loop operating as a motor or as a generator? If the direction of the field b is reversed in both figures, would the mode of operation change? How would you change fig. 1.6 to obtain the opposite mode of operation? How would you change fig. 1.7 to obtain the opposite mode of operation?

1.11 Repeat question 1.10 for figs. 1.9a and 1.13. Repeat it again for figs. 1.22 and 1.23.

1.12 For the developed diagram of the 4-pole machine shown in fig. 1.24a:

a. The coils on the poles are called field coils. They should be connected in series; draw the necessary interconnections between them.

b. Indicate on the diagram suitable directions for motion, currents, emfs, and forces in the armature conductors for motor operation.

c. Repeat part b for generator operation.

1.13 The armature of a 6-pole dc motor has 24 slots and rotates in the clockwise direction. Draw a cross-section of the machine showing(a)north and south poles, (b)all direct and quadrature axes, (c)a pole pitch, (d)flux paths, (e)field coils, with their interconnections and current directions, (f)armature conductors with current and emf directions, (g)directions of torque and rotation.

1.14 Repeat question 1.13 for a 4-pole generator whose armature has 16 slots and rotates in the counterclockwise direction.

1.15 The armature of a 10-pole dc machine has a diameter of 48 cm and rotates at 570 rpm. Find (a)the speed in rps (revolutions per second)(b)the speed in meters per second, (c)the angular speed, (d)the time it takes to complete one revolution, (e)the time it takes to complete one electrical cycle, (f)the time it takes to move one pole pitch, (g)the distance a point on the armature surface covers in one second.

1.16 In fig. 1.37c, let α denote the angle between the inclined surface and the horizontal plane, and β the angle between the applied force f_a and the inclined surface. Use the symbols defined on the figure, together with α and β, in the following requirements:

a. Write out the force equation along the axis parallel to the surface.

b. Write out the force equation along the axis perpendicular to the surface.

c. Assuming that the body moves up the inclined surface at a constant speed u, what is the energy lost by the agent applying the force $f_a$in a time interval Δt? Where does this energy go? What is the power exerted by the moving agent?

d. where does the lost energy in part c go if the speed changes as the body moves up the incline?

1.8 APPENDIX :Review of mechanics

This appendix reviews some basic principles from mechanics which are important to the study of electrical machines. First, we consider rectilinear motion (ie motion in a straight line), then modify our equations for rotary motion.

1.8.1 Rectilinear Motion

Consider a body of mass m moving with a velocity u as shown in fig. 1. 31a.From Newton's law, we have

f_r =ma=m du/dt newtons (1.37)

where f_r is the resultant force on the body, and a =du/dt is its acceleration, ie rate of change of speed. To understand the implications of eqn. 1.37, let us study the simple case of fig. 1.37b which shows a body moving along a horizontal surface under the influence of a horizontal applied force f_a. The friction force between body and surface, f_f, tends to oppose motion, so that the resultant force on the body is given by

$f_r = f_a - f_f$ (1. 38)

Here we have three possibilities:

$f_a > f_f \rightarrow f_r > 0 \rightarrow a > 0$ →increasing speed; (1.39a)

$f_a = f_f \rightarrow f_r = 0 \rightarrow a = 0$ →constant speed; (1.39b)

$f_a < f_f \rightarrow f_r < 0 \rightarrow a < 0$ →decreasing speed. (1.39c)

Note in particular the case given in 1.39b :if the resultant force is zero, motion is at constant speed; conversely, if a body is moving at a constant speed, we conclude that the resultant force on it is zero, ie the forces acting on it are balanced. Of course, a body at standstill will not start to move unless the applied force f_a exceeds the friction force f_f to produce the positive acceleration necessary to raise the speed from zero; once the body is in motion, f_a can be decreased to become equal to f_f to keep speed constant.If f_a is decreased further, there will be deceleration (ie negative acceleration)so that the body will slow down, possibly until it stops.

Fig. 1.37c shows a more complicated situation where the body is moving along an inclined surface. The forces have to be resolved along two axes, one parallel to the surface, and the other perpendicular to it. The parallel component of the applied force f_a is opposed by the friction force f_f and the parallel component of the weight mg, where g is the gravitational acceleration.

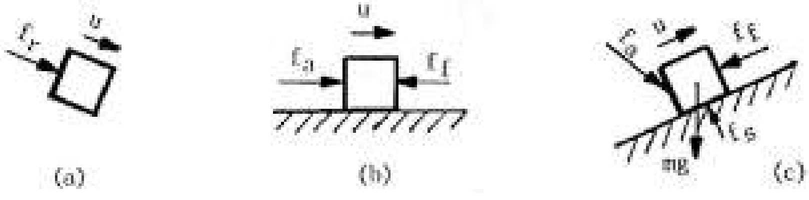


Fig. 1.37 Forces acting un bodies in rectilinear motion.

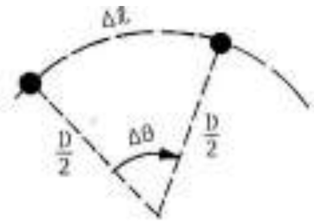


Fig. 1.38 A particle in rotary motion.

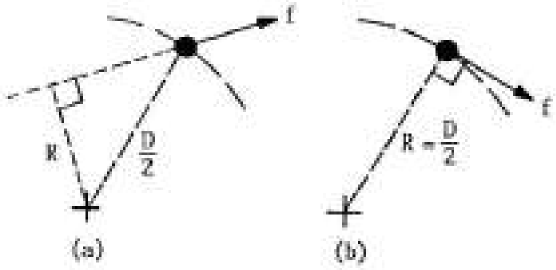


Fig. 1.39 The force arm.

The angular displacement between two points on the circular path is Δθ radians, fig. 1.38. The corresponding displacement in meters along the arc is

$$\Delta\ell = \frac{D}{2}\Delta\theta \qquad (1.\ 45)$$

The angular speed is ω,=dθ/dt radians/second, and the angular acceleration is α =dω/dt radians/second/second.

Eqn. 1. 37 is replaced by

$$\tau_r = J\alpha = J\frac{d\omega}{dt} \qquad (1.46)$$

where J is the moment of inertia of the rotating body in Kgm^2 Eqn. 1.46 can be understood by analogy with eqn. 1.37. The resultant torque T_r produces all acceleration, ie a change in speed; if T_r is in the direction of motion, the acceleration is positive, and the angular speed increases; if T_r opposes motion, the acceleration is negative, ie deceleration, and the angular speed decreases. If, however, the resultant torque is zero, then the acceleration is zero, and speed is constant; conversely, if speed is constant, we can conclude that the resultant torque is zero, ie the torques acting on the body are balanced. For the most part of our study of dc machines, we shall assume rotation to be at constant speed, which is called the steady state; the torques acting on the shaft, which include the torque developed electromagnetically, the friction torque, the load torque, etc, are in equilibrium. The dynamic, or transient, state occurs when one or more of these torque increases or decrease; the balance is disturbed, and there will be a momentary change in speed until a new steady state, at a different speed, is reached.

The work done in moving a body is given in eqn. 1.40. In rotary motion, the displacement is along the arc, so that eqn. 1.45 can be substituted for Δℓ:

$$\Delta W = f\frac{D}{2}\Delta\theta$$

The force is along motion, ie tangential to the path as in fig. 1.39b, in which case the torque is given by eqn. 1.44 so that

$$\Delta W = \tau\Delta\theta \quad (1.47)$$

The power is the rate of doing work, or rate of change of energy, as in eqn. 1.41, and for rotary motion, we get

$$p = \frac{\tau\Delta\theta}{\Delta t} = \tau\frac{\Delta\theta}{\Delta t} = \tau\omega \quad (1.48)$$

Conservation of energy and power still apply as discussed in section 1.8.1. Answers:

1.1 a.E= 38.4 (-y), V=28.4, p_c =76.8, G; b. 38.4 (y), f = 48 (-x), 76.8, M; c. 2 (-z),5(x), 225(-z),450; d. 4.44(-x),4.5(-x),20,M; e b=. 250(-y),u= 1.12(-z),E=70, G; f. 0, 0.96 (-y), 0, X.

1.2 0.1885 m, 45 deg mech, 0.785 rad mech, 180 deg elec, 3.1416 rad elec, 9 slots,140degelec, 8.75degmech, 0.1466m, 3.67cm.

1.3 2.09 cm,5 deg mech, 20 deg elec.

1.6 267mWb, 1.275 T, 26.7V, 4.59Nm, 240W.

1.7 36.4 V, 7.25A, 264W, 7-8W.

1.8 835mT, 0.75mWb, 23.87mNm, 3.0W.

1.9 a. 0; b. 7.96Nm, -7.96Nm; c. 0, 20W, 20W; d. G (unloaded), M, G;e. T =79.6 -19.9n.

1.15 9.5 rps, 14.3 m/s, 59.7 rad/s, 105 ms, 21 ms, 10.5 ms, 1.43 m.

1.16 a. $f_a\cos(\beta) - f_f - mg\sin(\alpha) = rna$; b. $f_a\sin(\beta) + mg\cos(\alpha) - f_s = 0$;c. $f_a\cos(\beta)u\,\Delta t$, $f_a u\cos(\beta)$

CHAPTER 2
CONSTRUCTION

The essential parts of the dc machine are the poles, where the main field is produced, and the armature conductors, which interact with the field to produce emf and torque. But the actual construction of the machine involves other parts for mechanical support, completion of electric and magnetic circuits, insulation of conductors, etc. This chapter shows how dc machines are constructed, and explains some important factors that influence their design.

2.1 Materials

In general, electrical machines are composed of electrical circuits coupled through magnetic circuits. The dc machine has basically two electric circuits, the field coils and the armature coils, coupled through the magnetic circuit.

Electric circuits must have low resistance to reduce voltage drop and I^2R losses in conductors, and the material most widely used is copper. Thus, field coils, armature coils, and commutator segments are all made from copper. The resistance of copper conductors is discussed in section 1.7.3.

To obtain strong magnetic fields, magnetic circuits are made from iron, which has a high permeability. The permeability is defined as the ratio of the magnetic flux density B to the magnetic field intensity H producing it:

$$\mu = B / H \quad \text{henries/meter} \qquad (2.1)$$

It is useful to think of H as an applied quantity, and B as the resulting quantity which depends not only on the applied H, but also on the permeability of the medium:

$$B = \mu H \qquad (2.2)$$

In principle, the permeability of a given material can be measured as illustrated in fig. 2.1. A ring specimen of the material is made, and a wire is wound around it. Current is passed in the wire to produce flux in the core(applied and resulting quantities respectively). The applied magneto-motive force is defined by mmf $= NI = H\ell$, so that

$$H = NI / \ell \quad \text{Amperes/meter} \qquad (2.3)$$

Where N is the number of turns, and ℓ is the mean length of the, core. The flux density is obtained from

$$B = \Phi / s \qquad (2.4)$$

Where Φ is the flux measured in the core, and s is the cross-sectional area of the core. With Hand B determined from eqns. 2.3 and 2.4, the permeability μ is obtained from eqn. 2.1. For air and copper, the permeability is constant and equal to the free space permeability

$\mu_0 = 4 \pi X 10^{-7}$ h /m.

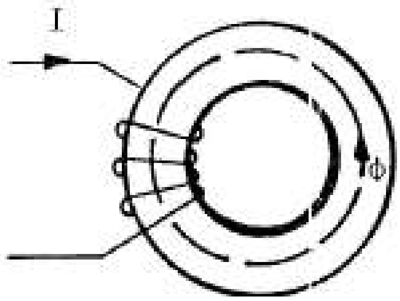


Fig. 2.1 Measurement of the B-H curve using a ring specimen of the material.

The permeability of iron is not constant because the relationship between B and H is not linear, fig. 2.2. Initially, at low field, the B-H curve has a sharp slope, ie a very high permeability; but as H is increased, the curve goes through a bend, called the 'knee' of the curve, and then enters saturation where even large increases in H produce only negligible increases in B; that is, the flux density B is almost constant when the field is strong enough to saturate the iron. Commercial steels are alloys where the types of additives and their percentages, as well as the manufacturing process, determine the magnetic characteristics, ie the actual B-H curve.

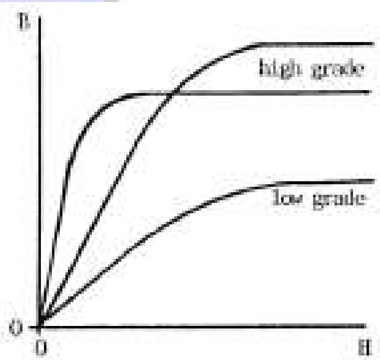


Fig. 2.2 Typical B-H curves of commercial steels.

High grade steels are characterized by a high initial permeability ($\mu_r \geq 1000$), and a high saturated flux density (up to 2 Tesla); they are used for magnetic cores (armature and poles). Low grade steels are used in constructional parts for mechanical support. It is to be noted that different types of steel not only have different magnetic characteristics, but also different mechanical characteristics; of course, all parts should be strong enough to withstand the stresses they are subjected to in addition to copper and iron, electrical machines employ insulating materials to separate cupper conductors from each other and from iron parts.

Different types of insulation are used, including paper and synthetic tapes which may be impregnated in resins etc; commutator segments are insulated from each other by solid mica insulators. A main feature of all insulating materials is their sensitivity to temperature rise : beyond a certain temperature different for each type of material, the insulator breaks down allowing current to flow between the conductors it is supposed to separate. Such 'short circuits' are the most common type of fault that occurs in electrical machines.

2.2 Temperature Rise

When an electrical machine is working, it tends to heat up because some of the input energy is converted to heat energy instead of useful output; that is, the process or electromechanical power conversions accompanied by some loss in power. Obviously, such power loss is undesirable because it represents waste. But more importantly, it is undesirable because the heat generated raises the temperature of the various parts of the machine; temperature rise is most critical to insulating materials :their insulation deteriorates at around 100 °C, and ultimately breaks down causing short circuits and possibly total machine failure.

Losses cannot be eliminated, but machines are designed to make them as small as possible, and also to limit the temperature rise by efficient heat dissipation and ventilation. As we shall see later, certain features of the practical construction of machines are intended specifically to limit the temperature rise.

Losses are generated in a number of ways. We have already seen in section1.7 that current flow in conductors produces I^2R loss; this is called copper loss, and occurs in armature coils as well as field coils.

The rotation of the armature produces mechanical losses due to friction. Losses also occur in iron cores due to hysteresis and eddy currents. Hysteresis loss can be kept small by using steels whose hysteresis loops are thin since the loss is proportional to the area enclosed by the loop. Eddy currents in the armature iron are driven by the emf induced due to rotation in the magnetic field; current flow, of course, produces I^2R loss in the iron itself. The eddy current loss is reduced by effectively increasing the iron resistance in the path of current flow, which is done by constructing cores using stacks of steel laminations instead of solid steel. Laminations are sheets 0.5 -1.0 mm thick, with their surfaces electrically insulated using suitable coatings; the laminations are stacked and pressed together to form the required core, fig. 2.3. The laminations are arranged so that the insulated interfaces are in the path of the induced emf and current, but not in the path of the magnetic flux so as not to increase the reluctance; that is, the flux paths should be in the plane of the laminations.

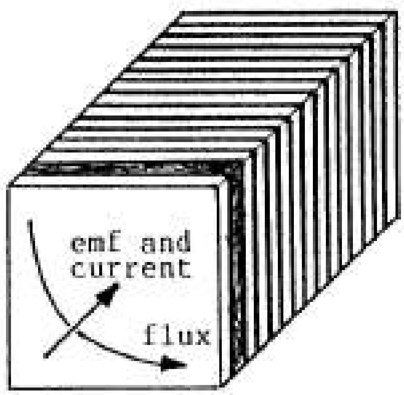


Fig. 2.3 Laminated iron part.

The total length of the armature block includes both iron and insulation; the effective length of the armature includes only the iron, because the insulation as the permeability of free space μ_0The stacking factor is defined as the ratio of the useful effective length to the total length:

$$\text{Stacking factor} = \frac{effectivelength}{totallength}$$

The stacking factor ranges from 90 %to 97 .%

Copper losses and core losses generate heat inside conductors and iron cores respectively. The heat is removed by contact with air at the surfaces of the material blocks. When the machine is working, the temperature rises until the rate of heat removal is equal to the rate of heat generation; the temperature then remains constant. The machine is designed so that the various materials can withstand their steady-state temperatures; that is, the temperature in any part of the machine does not rise beyond the permissible value for the materials in that part. To improve the rate of heat removal, and hence limit temperature rise, the contact surfaces with the cooling air aloe increased, for example by drilling ducts through iron cores, or attaching metal fins to external surfaces; note that the external surfaces or copper conductors are in immediate contact with insulating materials which are particularly sensitive to temperature rise. The machine design should also provide for good ventilation, that is the flow of the heated air outside the machine, and its replacement by fresh cooling air.

2.3 Machine Rating

DC machines are manufactured in many sizes, and operate on different voltages with different current and speeds. For each machine there are, in principle, specified values of voltage, current, speed, torque, power, etc. which are called the 'rated values', ie rated voltage, rated current., etc. Usually there is a small p1ate attached to the cover of the machine on which the rated values are written, together with additional information such as type of machine, manufacturer name, serial number, etc; this is called the nameplate of the machine. The specification of rated values does not mean that the machine has to be operated at these values all the time; for example, a machine rated at1200 rpm may be rotated at 1000 rpm, 1500 rpm, or even 2000 rpm. The concept of rating may be understood as a sort of contract between the manufacturer and the user :if the user operates the machine according to its rating, then the manufacturer guarantees that it will give satisfactory service over its specified lifetime, say 20 or 30 years; if, however, the user operates the machine outside its rating, then its useful life is shortened. In severe cases where the user attempts to operate the machine far outside its rating (for example, rotating a 1200 rpm machine at 5000 rpm), the machine may break down.

The design details of the machine are determined by the intended rated values. Current determines conductor cross-sectional areas :higher currents require thick conductors to limit copper loss and to increase surface area for heat exchange. Voltage determines core size recalling eqn. 1.13c, the average air gap flux density B does not exceed 0.8 T in practice (corresponding to 2 Tin the core -see section 2.1); to increase the voltage for a given speed ω and a given number of turns N, the designer has to increase the armature diameter D, its length L, or both -see fig. 1.26. The voltage also determines the amount of insulation between conductors. Speed determines mechanical aspects of the design: the armature and its parts should withstand the centrifugal forces caused by rotation, and vibration should be as small as possible.

Insulating materials determine the maximum allowable temperature rise, which in turn, determines the limits on the allowable losses in the various parts of the machine and the measures taken to aid heat dissipation.

Operating the machine according to its rating means that the electrical, mechanical, and thermal stresses in all parts of the machine are within acceptable values for the materials used. Operating the machine, a little bit outside its rating for short intervals has little effect, although it may shorten the machine's lifetime slightly. Operating the machine far outside its rating for long intervals can cause severe deterioration, or damage to parts that have been driven outside their acceptable limits (for example, insulation breakdown due to severe overvoltage or temperature rise, mechanical breakdown due to severe over-speed, etc). Implicit in this discussion is an economic factor: better materials and parts can withstand higher stresses, but are, of course, more expensive.

The nameplate of the machine usual1y gives its continuous rating; that is, the machine can be operated continuously at the rated values listed. The current and power rated va1ues correspond to full load applied continuously; if the load on the machine is reduced, the current and power will be less than their rated values, which is acceptable. In certain cases, the rating is given as a range of values; for example, motors required to operate at different speeds have the allowable speed range specified, say 1000-2000 rpm (the 1ower limit is determined by the requirements of ventilation). For certain applications the nameplate gives short-time rating; for example, a generator may have a continuous current rating of 5 A, but a short-time rating of, say, 10 A for 2 minutes; this means that the temperature does not rise to a dangerous level in 2 minutes. Intermittent rating is similar to short-time rating, but a1so gives the minimum time interva1 allowed between successive applications of the high value; this interval allows the machine to cool down.

On the electrical side, the power of the machine is the product of voltage and current. As we have seen, higher rated voltages require larger cores, while higher currents require thicker conductors. In general, then, higher power ratings require bigger machines; that is, the physical overall size of machines tends to increase with their power ratings.

Machines are usually manufactured in standard frame sizes to facilitate their installation. They also come in different enclosures to suit various environmental conditions. For example, standard enclosures are for dry environments; however, if the machine is to be installed in a wet environment, the enclosure should protect the machine; manufacturers can supply machines that are drip-proof, splash-proof, or even totally submersible.

To select a suitable machine for a given application, the user must know the requirements of his system, such as the available voltage supply, the required torque and speed, the environmental conditions, etc. He then studies manufacturers' proposals, and chooses the most economical machine that meets his specifications.

2.4 Main Parts of The Dc Machine

All rotating electrical machines have a stationary part called the stator, and a rotating part called the rotor; the two are separated by a small clearance, the air gap, to allow relative motion between them. In dc machines, the poles, where the field is produced, are placed on the stator, while the armature, whose coils interact with the field, is placed on the rotor. The dc machine also has a commutator attached to the rotor and rotates with it; the brushes are stationary, being attached to the stator. Fig. 2.4 shows the main constructional features of dc machines; actual machines can differ greatly in the details of construction.

2.4.1 Stator:

The cylinder enclosing the machine is called the yoke; the poles are fixed to it, usually by means of bolts. The yoke provides a return path for the magnetic flux. Each pole is composed of a core, around which the field coils are placed, and a shoe, which is the part facing the armature. The pole shoe has tips extending beyond the core which serve to improve the flux density distribution in the air gap, fig. 2.5.

The pole tips increase the effective area of the air gap, thus reducing its reluctance to the magnetic flux. The tips also provide mechanical support for the field coils. The pole shoes are usually made from laminated steel because the rotating armature slots and teeth cause the air gap field to fluctuate, resulting in eddy current losses in the pole shoes. The pole care mayor may not be laminated, but in modern motors operating from solid state drives, even the yoke is laminated.

The field coils provide the main working flux. In certain machines, there are two coils on each pole :one coil has many turns of fine wire, and the other coil has few turns of thick wire. Identical coils are placed on all poles, and the coils of each set are connected in series. Permanent magnet machines have no field coils; the poles are permanent magnets, with the flux path completed through the yoke.

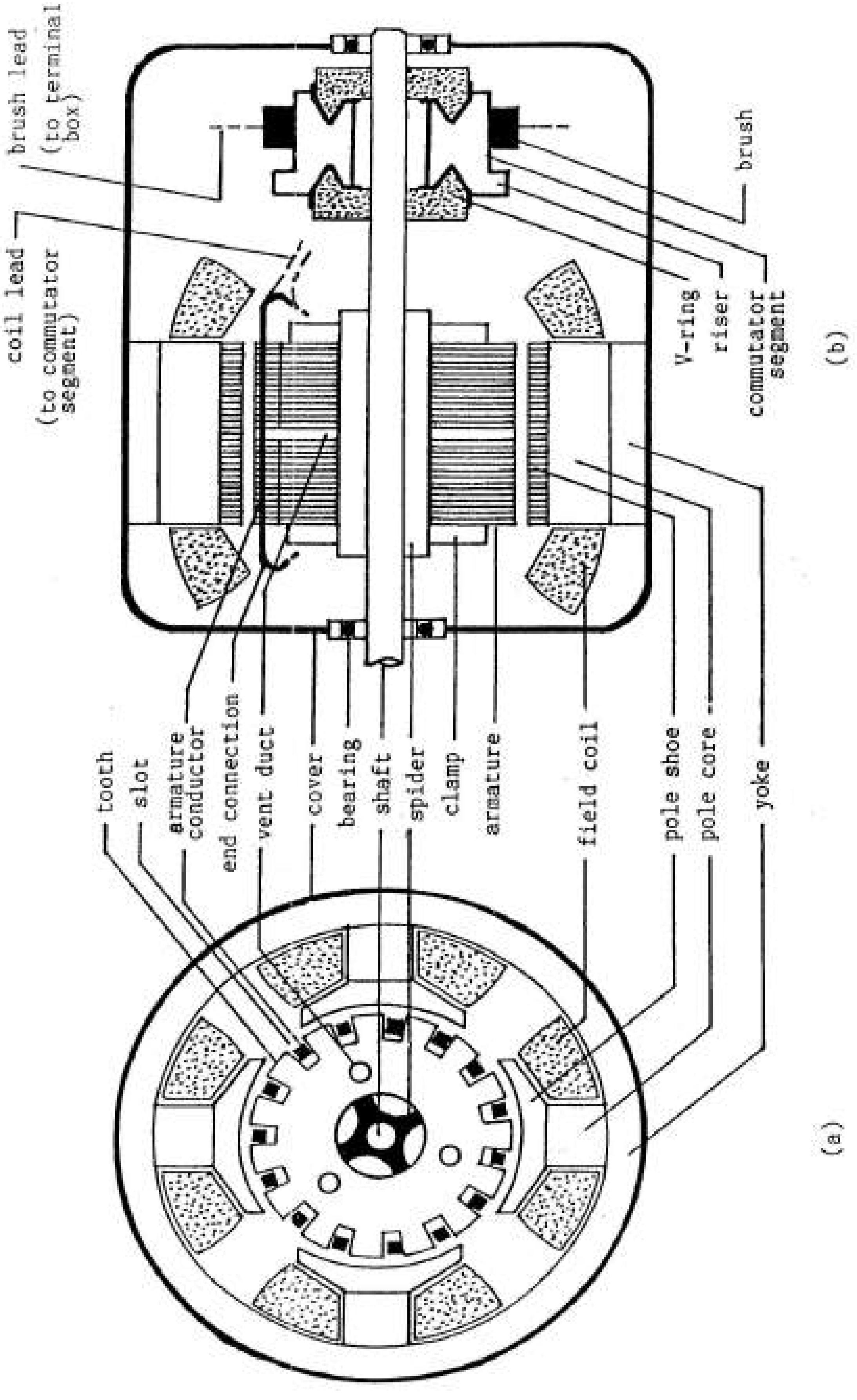


Fig. 2.4 Typical dc machine :(a)cross section; (b)longitudinal section

2.4.2 Rotor

The armature is made from circular laminations, punched with holes for slots, vent ducts, and a central bore for the shaft. The laminations are stacked together, and clamped to form a rigid cylinder. In small machines, the armature is mounted directly on the shaft; in large machines, the armature is mounted on a spider, and the spider is mounted on the shaft. The spider is a mechanical fixture, and does not need to be made from high grade magnetic steel. The shaft rotates in bearings or bushings fitted to the machine cover which is attached to the stator. Axial vent ducts are formed by holes punched in the laminations, fig. 2.4a; radial ventilation in large machines is achieved by stacking the armature laminations in packets, with space between packets for air flow, fig. 2.4b.

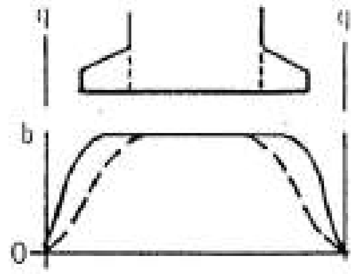


Fig. 2.5 Improvement of air gap flux density distribution due to pole

The armature coils are placed in slots around the periphery; the end connections (see fig. 1.5) extend outside the iron cylinder of the armature, fig. 2.4b. The coil leads are taken to the commutator, and the terminals are soldered to its segments. Typical slot shapes are shown in fig. 2.6; the iron between adjacent slots forms a 'tooth'. The fixing wedge helps keep the conductors in position in the slot against the centrifugal forces arising from rotation. In large machines, where currents are high, the armature coils are made from thick and rigid copper strap, fig. 2.6a; the coils are formed outside the armature, then placed in the slots. In small machines, the currents are small, and the coils are made from relatively thin flexible wire, fig. 2.6b; they may be pre-formed outside the slots, or they may be wound directly on the armature. The insulation occupies a larger proportion of the slot area in small machines than in big machines. The slots may be straight as in fig. 2.7a, or they may be skewed as in fig. 2.7b. Skewing is done by a slight rotation of consecutive laminations during stacking; the total skew between the two ends of the armature is usually one slot. Skewing helps reduce noise and vibration by allowing the active conductors of the armature to enter smoothly under

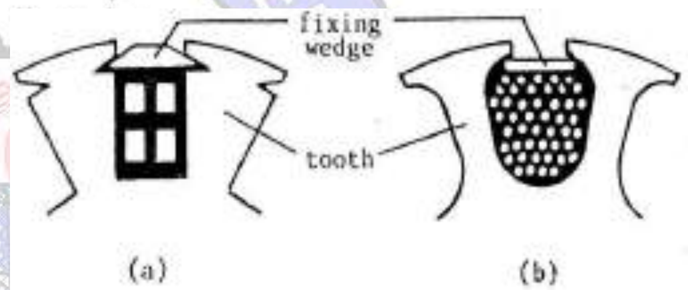


Fig. 2.6 Typical armature slots.

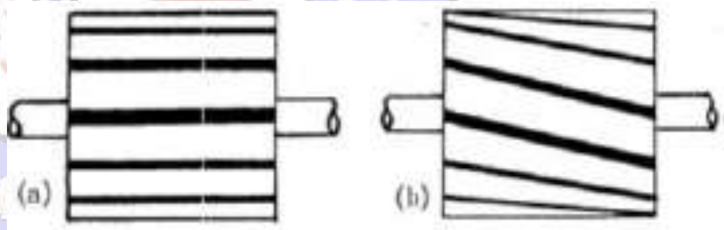


Fig. 2.7 Armature side view with (a) straight slots, and (b)skewed slots.

the poles during rotation; however, it increases the length of the coil, and hence its resistance, without any increase in induced emf or torque.

2.4.3 Air gap

The air gap is the space between the pole face and the armature surface; such clearance is necessary to allow relative motion between the two. The air gap also helps in heat removal from the armature surface, and in ventilation. However, it introduces a high reluctance in the path of the magnetic flux, and is therefore made as short as possible, typically 0.5-5.0 mm. The armature surface and the pole shoe facing it require precise machining to ensure symmetry and avoid vibration.

2.4.4 Commutator

The commutator is mounted on the shaft, so that it rotates with the armature, fig. 2.8. It is made up of copper segments insulated from each other and forming a cylinder. The insulation between segments is mica or micanite of thickness 0.5 -1.0 mm; it is undercut below the copper surface, also by0.5 -1.0 mm. The number of segments is equal to the number of coils in the armature; the terminals of each coil are soldered to two different segments.

Each segment has a riser in which the coil terminals are placed and soldered. The segments of the commutator are held together by means of two V-rings. Which are mounted on the shaft, fig. 2.4b; the V-rings clamp the segments, but are electrically insulated from them. The brushes are made from carbon or graphite; they are mounted in stationary holders, with spring pressure to maintain good electrical contact with the rotating segments. Sparking and friction between brushes and commutator cause the brushes to wear out, so that they should be replaced

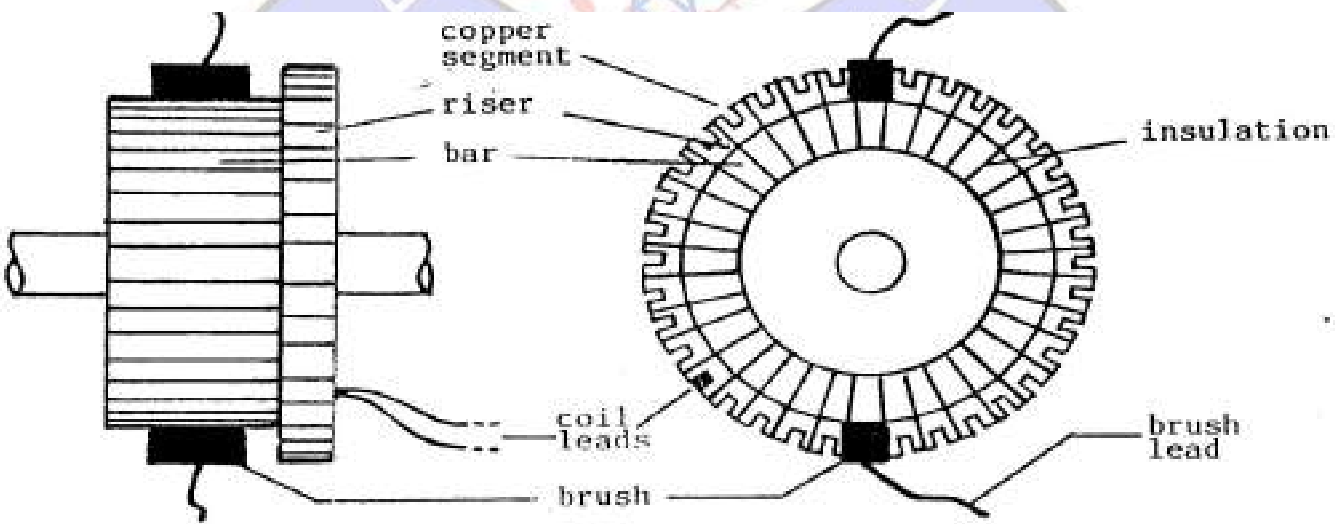


Fig. 2.8 Commutator and brushes

regularly. The commutator also requires regular maintenance to remove carbon filings and dirt that tend to accumulate on it, especially between segments.

2.5 Miscellaneous items

Section 2.4 explained the main parts of the dc machine; in this section we look at some of the other items that may be found in the machine.

The machine is protected by a metal cover that gives it the overall shape of a cylinder. The cover mayor may not extend over the stator yoke, but it has caps that protect the commutator on the right of fig. 2.4b and the end connections on the left. The cover has windows to aid ventilation; it also has removable plates in the region of the commutator to allow servicing of commutator and brushes.

The shaft carries all the rotating parts of the machine, and transmits torque into the machine or out of it; these mechanical stresses determine the diameter of the shaft. At the two ends of the machine, the shaft rests in bearings or bushings fixed to the cover; friction in these mountings is minimized by greasing. At one end, the shaft extends outside the cover to allow external coupling, which may be through pulley and belt, through gears, or direct. In many machines, a fan is mounted on the shaft to improve ventilation.

Heavy machines usually have one or more lifting eyes fixed to the yoke. The machine may also have a base, or some other fixture, for installation.

The leads from the main electrical circuits inside the machine are brought to a terminal box fixed to the outside of the machine to allow external connections. In the box, there are two terminals for the brushes, which provide the connections to the armature; there are also two terminals for each set of field coils.

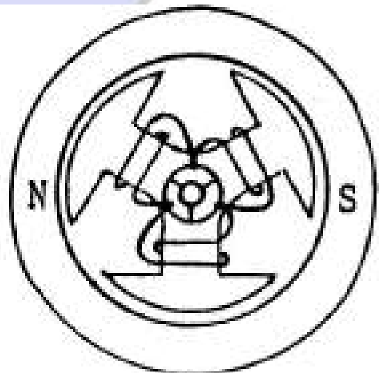


Fig. 2.9 Miniature permanent-magnet dc motor (actual size).

The armature of very small dc machines, called miniature machines, may have only three slots and three teeth as shown in fig. 2.9; the slot pitch (see exercise 1.3) is two-thirds the pole pitch. The commutator has three segments, and the terminals of each coil are soldered to adjacent segments. The cover at the commutator end carries a pair of metal-carbon brushes on spring strips which are also the terminals. The field in fig. 2.9is an annular 2-pole permanent magnet. Such miniature motors run on small batteries, and their power rating is typically around five watts; they are used in toys, tape recorders, etc. Miniature motors similar to that shown in fig. 2.9 are also constructed with five slots.

2.6 Exercises

2.1 List the main types of materials found in dc machines, and explain why each is used.

2.2 Discuss the effect of saturation on the permeability of iron.

2.3 Why are armature coils not made from silver? from iron?

+2.4 a. Explain how hysteresis arises in the armature iron.

b. The iron in pole shoes undergoes minor hysteresis loops; discuss.

2.5 Explain why the effective length is less than the total length of a laminated armature. Which value should be used for L in eqns. 1.13and 1.24? Which value should be used in resistance calculations?

+2.6 Explain why the armature laminations are clamped together without any bolts going tight through them.

2.7 Does the use of laminations reduce hysteresis loss?

2.8 Sketch a laminated iron cylinder, and indicate the directions of eddy current flow and magnetic flux. Why are the laminations arranged to be parallel to the flux paths and not perpendicular to them?

2.9 Laminating iron parts increases their resistance, and hence reduces eddy currents. Reducing the emf induced in iron also reduces eddy currents. How can the emf in iron be reduced? and why is this not considered to be a good method for reducing eddy currents?

2.10 a. A dc generator is rated at 150 V and 9 KW. What is the rated current?

b. A 25 hp motor is rated at 1200 rpm. What is the rated torque ?

2.11 A machine is rated at 1500 rpm. What damage is to be expected if it is rotated at 3000 rpm? at 300 rpm ?

2.12 List in sequence the parts of the machine in the closed path of magnetic flux.

+2.13 With reference to fig. 2.6, discuss the advantages and disadvantages of

a. using metallic fixing wedges; b. making the teeth thin.

+2.14 Skewing the armature increase the conductor length in the slot. Why then does skewing not increase emf and torque, both of which are proportional to the active length, eqns. 1.13 and 1.24 ?

2.15 Why does insulation occupy a larger fraction of the slot cross-sectional area in smaller machines?

CHAPTER 3
ARMATURE WINDING

In chapter 1 we saw that the armature has a number of coils displaced from each other in space. The coils are connected to each other at the commutator segments to form the armature winding. This chapter explains the distribution of coils in slots, the connection of coil terminals to commutator segments, and the resultant equivalent circuit of the armature.

3.1 Winding details

The coil span is the distance, along the armature surface, between the two sides of the coil, fig. 3.1; it is equal to a pole pitch so that when one coil side is under a north pole, the other coil side is under a south pole, and hence their emfs add, section 1.5. The coil span can be measured in units of length, in mechanical degrees, or in electrical degrees; it can also be measured in slot pitches, where a slot pitch is the distance from the center of one slot to the center of the next slot. Thus in fig. 3.1, the coil span is four slot pitches, or four slots for short; clearly, the coil spans an integer number of slots.

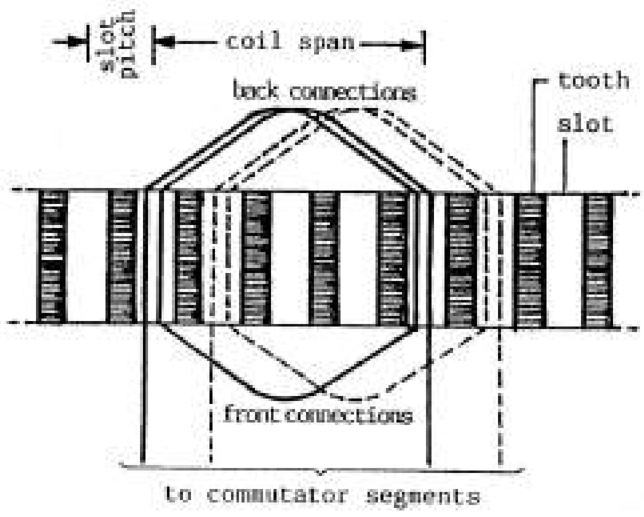


Fig. 3.1 Developed side view of armature showing locations of two coils.

Let

N =number of turns per coil;

C =total-number of armature coils;

Z= total number of armature conductors.

Each turn has 2 active conductors, so that each coil has2N active conductors; then

Z =2NC (3.1)

It should also be clear that the total number of coil sides is 2C =*Z/N*, and that the total number of armature loops or turns is NC.

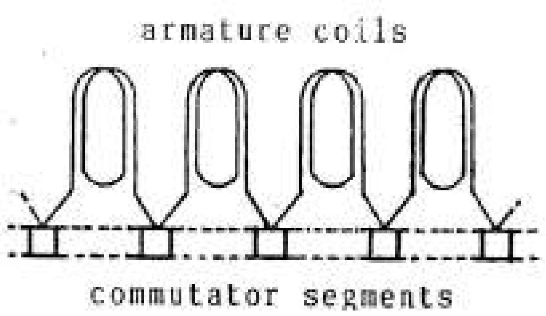


Fig. 3.2 Interconnection of armature coils on commutator segments.

The two terminals of each coil are soldered to the risers of two different commutator segments. Moreover, the riser of each segment has two terminals, coming from different coils, soldered to it. Since there are C coils, there are 2C coil terminals, and hence 2C /2 =C commutator segments; that is

Number of commutator segments =number of armature coils =C.

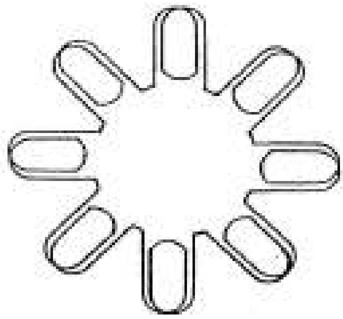


Fig. 3.3 Armature coils are connected to form a closed winding.

All the coils on the armature are connected together, on the commutator, to form a single closed winding :starting with any coil, its end is connected to the beginning of a second coil; the end of the second coil is connected to the beginning of a third coil, and so on, as in fig. 3.2. this is repeated until the last coil is reached :its end is connected to the beginning of the first coil, and the winding is closed, fig. 3.3. The diagrams in figs. 3.2 and 3.3 do not show which coils are connected to which segments; the sequence of interconnected coils and segments will be explained in section 3.2. What is important to understand here is that the armature coils form a closed winding, and that the interconnections between coils are made on the commutator segments. Note in particular that the closed armature winding, as shown in fig. 3.3, has no terminals, and we cannot say which coils are in series or parallel; in fact, it is the brushes, contacting the commutator segments, which provide the terminals of the winding, and divide the coils into series and parallel groups.

Armature coils are arranged in two layers :for each coil, one side is placed in the top half of its slot (top layer), and the other side is placed in the bottom half of its slot (bottom, layer); the bottom half of the first slot and the top half of the second slot are filled by other coil sides, fig. 3.4.

Two-layer windings allow coil end connections, in the front and the back, to be arranged in a regular manner; otherwise, the end-connections of adjacent coils will be in each other’s way. All the coils in a two-layer winding can be made identical, which simplifies manufacture, and results in a symmetric and compact armature winding. Small machines usually have one coil side per slot per layer, and hence two coil sides per slot, fig. 3.4. Large machines may have more coil sides per slot; for example, in fig. 3.5 there are three coil sides per slot per layer, and hence six coil sides per slot. Because there are two layers, the number of coil sides per slot is always an even number :two in fig. 3.4, and six in fig. 3.5.

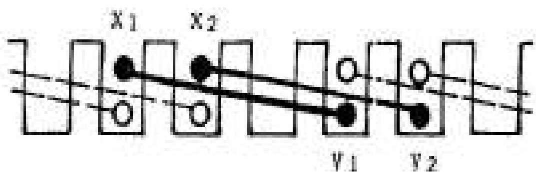


Fig. 3.4 Two-layer winding.

In fig. 3.5, x_1 and y_1 are the two sides of one coil; clearly, the coil has 4 turns, and each coil side has 4 conductors. The two terminals of the coil are soldered to commutator segments. Similarly, x_2 and y_2 are the two sides of another coil which has 4 turns, and whose terminals are also soldered to commutator segments.

Fig. 3.5 Two-layer winding with 3 coil sides per slot per layer.

Let S = number of armature slots;

M= number of coil sides per slot per layer.

In fig. 3.5, m=3; also, N=4, so that the number of conductors per slot per layer is 3x4 = 12, and the number of conductors per slot is 2x12 =24. In general, there are mN conductors/slot/layer, and hence 2mN conductors/slot.Clearly, then.

$$Z = 2mNS \tag{3.2}$$

Substituting for Z from eqn. 3.1, we find

$$C = mS; \tag{3.3}$$

thus, the number of coils C is an integer multiple of the number of slots S. The two are equal when m =1, that is, for windings having one coil side per slot per layer, or two coil sides per slot, as in fig. 3.4.

In section 1.5, the pole pitch was defined as $\pi D/2p$, the distance on the armature surface corresponding to one pole; it was also shown how the pole pitch may be measured as a mechanical or electrical angle. It is often useful to measure the pole pitch by the number of slot pitches it covers :the armature has a total of S slots, so that to each pole there correspond S/2pslot pitches, or simply slots; thus

$$\text{pole pitch} = s/2p \text{ slot pitches, or slots.} \tag{3.4}$$

For example, if the armature of an 8-pole machine has 120 slots, the pole pitch is 120/8 =15 slot pitches, or slots. Recalling that the coil span should be equal to a pole pitch, it is also 15slots in this example. Now suppose that an 8-pole machine has 122 slots in the armature; the pole pitch is given by eqn. 3.4 as 122/8 =15.25 slot pitches, or slots. But the coil span, when measured in slots, has to be an integer because it represents the number of slots covered in moving from the first coil side to the second coil side, both of which are placed in slots; in this example, then, the coil span cannot be exactly equal to the pole pitch. In such cases, the coils are wound with a span just below, or just above, the pole pitch; that is, 15 slots or 16 slots in our example. Coils whose span is approximately, but not exactly, equal to the pole pitch are described as being chorded; the following terms are also in common use:

coil span =pole pitch → full-pitched coils

coil span < pole pitch → short-pitched coils

coil span > pole pitch → long-pitched coils

chorded coils

Thus, for a pole pitch of 15 slots, as in the first example above, the coils are full-pitched with a span of 15 slots. In the second example, with a pole pitch of 15.25 slots. the coils cannot be full-pitched; they may be short-pitched with 15 slots, or long-pitched with 16 slots. In all cases, the coil span should not differ too much from the pole pitch to ensure that when one coil side is under a north pole, the other coil side is under a south pole.

3.2 Some Numbers

C= total number of coils on armature;

N = number of turns in each coil;

S = total number of a slots in armature;

Z= total number of armature conductors; = 2NC.

$\frac{S}{2P}$ = total number of slots / poles, = pole pitch measured in slots.

NC =total number of loops;

2C= Z / N = total number of coil sides;

Z / S = 2NC / S = number of conductors / slots;

2C / S = number of coil sides / slot;

NC / S = number of conductors / slot/ layers;

C/ S = number of coil sides / slot / layer.

3.3 Winding schemes

In the last section, it was emphasized that the armature coils form a closed winding, fig. 3.3, and that the interconnections between coils are made on the commutator segments, fig. 3.2. This section explains the sequence of interconnected coils and segments, that is, which coils are soldered to which segments.

Recalling that the two terminals of each coil are soldered to two different commutator segments, let us define the commutator pitch as the distance, on the commutator, between the coil terminals. This distance need not be measured in units of length, but simply in number of segments advanced as we go from the segment of the first terminal to the segment of the second terminal; we shall use the symbol y_c for the commutator pitch, and it should be clear that y_c is always an integer number of segments.

There are two ways for interconnecting coils and segments, lap and wave, which will be explained in the following sections. Lap windings are easier to understand, and hence will be studied first; but it is wave windings that are more widely used. To keep the diagrams relatively clear, each coil is shown to have a single turn; moreover, the coil side lying in the top layer is shown in full-line, while the coil side lying in the bottom layer is shown dashed.

In certain diagrams, there are arrows on coil sides; these may represent the direction of induced emf, or the direction of current.

3.3.1 Lap winding

In the lap winding scheme, each coil is connected to the coil that lies immediately next to it on the armature. Such adjacent coils are in approximately the same position relative to the poles; that is, they see almost the same flux density. The terminals of each coil are soldered to adjacent commutator segments, so that the commutator pitch y_c is one. Fig. 3.6 shows that there are two ways of forming lap windings, progressive and retrogressive; the choice between these has no important effects. The commutator pitch is taken to be positive for progressive winding, and negative for retrogressive winding; thus, for lap windings in general

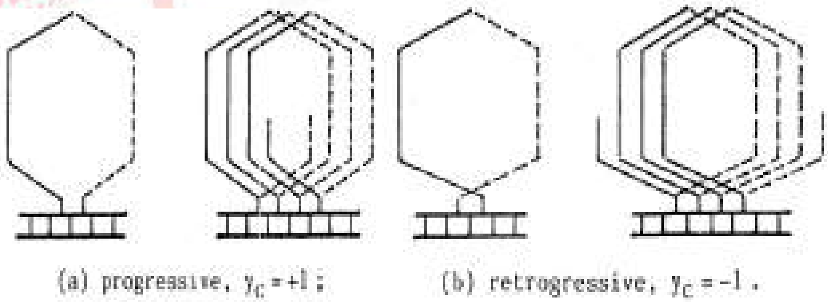


Fig. 3.6 Lap winding

$y_c = \pm l(\text{lap}).$ (3.5)

In both cases, the connection of the end of one coil to the beginning of the next coil is continued until the last coil, the Cth coil, is reached, whose end then closes on the beginning of the first coil.

As an example, let us consider a 4-pole armature with 22 coils (2p=4, C =22). For simplicity, we assume the armature has 22 slots, so that there are only two coil sidesin each slot, one in the top layer, and one in the bottom layer (m =1, and C=S in eqn. 3.3). The pole pitch is 22/4=5.5 slots, so that the coils cannot be full-pitched; they can be short-pitched with a coil span of 5 slots, or long-pitched with a coil span of 6 slots. In our example, we choose short-pitched coils, which have the advantage of shorter end-connections. The complete winding is shown in fig. 3.7 as a developed diagram. Let us first study the coils and commutator segments, both of which are on the rotor; that :is, they move together. The coils and segments are numbered sequentially such that the first terminal of coil 1 is soldered to segment 1; of course, any coil could have been chosen as number 1. The winding is progressive, $y_c+ = 1$, so that the second terminal of coil 1 is soldered to segment 2. Also soldered to segment 2 is the first terminal of coil 2, which lies immediately after coil 1 on the armature; the second terminal of coil 2is soldered to segment 3, on which coil 3 starts. You can thus follow the coils, one after the other, passing through their interconnections on the commutator segments. When you reach coil 22, you will see that it starts on segment 22, and ends on segment 1; that is, it closes on coil 1.

Consider now the slots. Although they are not drawn explicitly in the figure, their positions are indicated. Each slot contains two coil sides, one in the top layer, drawn in full-line, and one in the bottom-layer, drawn dashed.

For example, slot 1 has the first side of coil 1 in the upper layer, and the second side of coil 18 in the bottom layer. The coils are short-pitched with a coil span of 5 slots; thus coil 1, which has its first side in slot 1, has its second side in slot (6=1+5). Coil 20 has its first side in slot 20 and its second side in slot (3= 20+5-22). Each coil has its first side in the top layer (drawn in full-line), and its second side in the bottom layer (drawn dashed). Note that the two coil sides in each slot are at the same position, although in the diagram they appear to follow each other around the armature.

Consider, finally, the poles; these actually lie above the active conductors, but have been drawn as shown in fig. 3.7 for clarity. The brushes and quadrature axes positions are also indicated. The distance between consecutive q-axes is a pole pitch, and it is slightly greater than the coil span; compare, for example, the distance from q_4 to q_l with the distance from slot 11 to slot 16, which represents one coil span, short-pitched.

The poles, q-axes, and brushes are not part of the armature winding; they are stationary, while the armature is in motion. Assuming the armature is moving to the right, in a short while slot 1 will be in the position of slot 2, which will have moved to the position of slot 3, and so on; meanwhile, poles, q-axes, and brushes will remain in their positions. Now, we know from chapter 1that poles alternate in polarity :if P1 is north, then P2 is south, P3 north, and P4 south, and vice versa. The arrows on the active conductors may be taken to indicate the directions of current flow :all the arrows under each pole are in the same direction, and are opposite to the arrows under the next pole; we have already studied this in chapter 1 -see, for example, figs. 1.22 and 1.23.

Coils 6, 11,17, and 22 have no arrows on their active conductors :each of the coils is short-circuited by a brush at the instant shown in fig. 3.1; such coils are said to be “undergoing commutation”, ie their currents and emfs are being reversed. To understand this, consider, for example, coil 10; at the instant shown in fig. 3.7, coil 10 is active. But in a short while, coil 10will have moved to the position of coil 11, so that brush B_3 short-circuits it. A little while later, coil 10 will come out from the short, and it will take the position of coil12, so that its arrows will be reversed with respect to the arrows of coil 10 in fig. 3.7. This reversal occurs in each coil every time it passes through a brush short-circuit. But the brushes are placed in such positions that the coils they short-circuit are the ones which have their active sides at or near the q-axes; that is, the reversal occurs in each coil when it passes through a q-axis (ie when its two sides are at q-axes). The q-axis is therefore sometimes called the brush axis : although the brushes do not themselves lie there, the coils which are in direct contact with the brushes have their active sides; there. It is noted that the instantaneous emf induced in a coil passing through the q-axis is zero because the flux density b is zero there, fig. 1.24b.

It should now be clear that in fig. 3.7 the armature coils, slots :and commutator segments are in motion, while the poles, brushes, q-axes, as well as the arrows, are stationary. The arrows do not change their directions, although the conductors to which they are attached are changing all the time.

The closed winding formed by the coils is shown in fig. 3.8; the diagram shows the sequence with which the coils are connected to each other. The brushes divide the coils into four groups; for example, coils 1, 2, 3, 4, and 5 are in series between brushes B1 and B2; similarly, coils 10, 9, 8, and 7 are in series between brushes B3 and B2. Each group of series coils provides a current path between two brushes, and the inter-brush connections connect the four paths in parallel with each other, as shown in fig. 3.9.

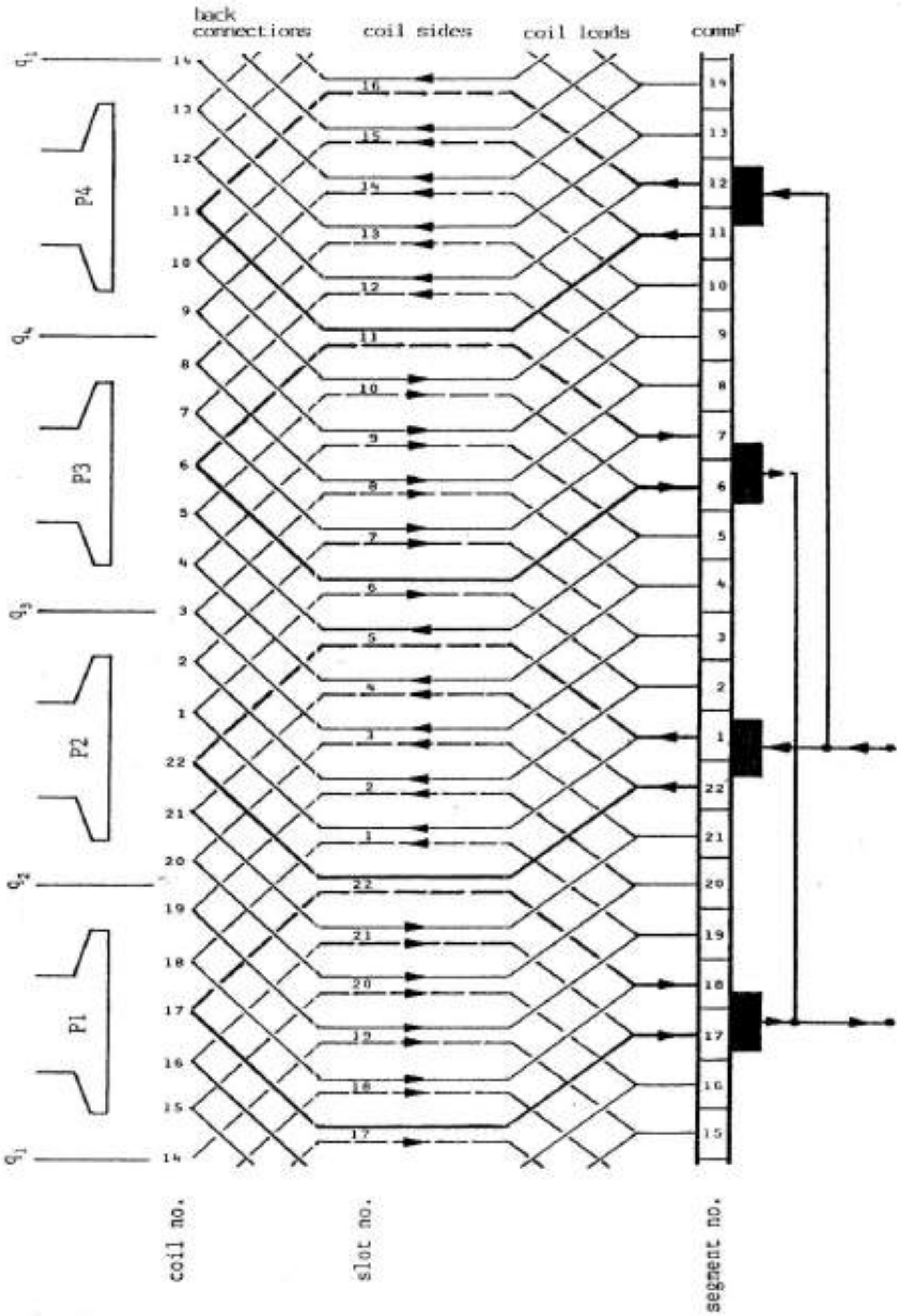


Fig.3.7 Simple lap winding :developed diagram for 4 poles and 22 armature coils

To understand these connections, let us follow some of the paths :the current entering into the machine divides between brushes B1 and B3; the current into B1 divides between two paths, coils 1-2-3 -4-5 to the right, and coils 21-20-19-18 to the left; the right path terminates on B2, and the left path on B4; but B2 and B4 are connected externally to each other; therefore, these two paths are in parallel with each other between brush Bl and brushes B2/B4.

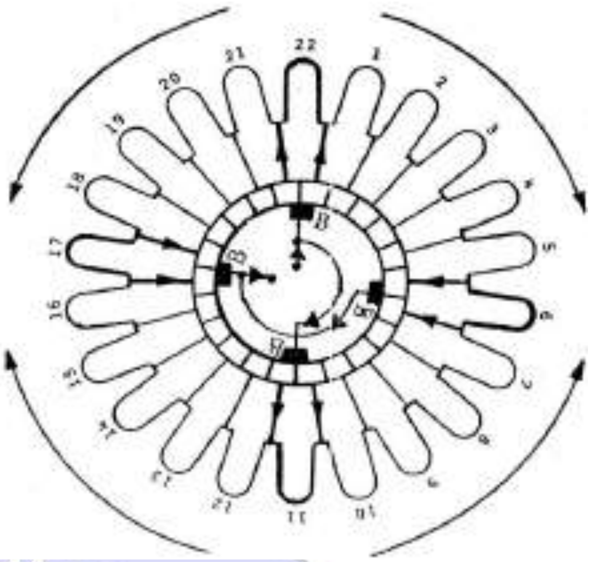


Fig. 3.8 Simple lap winding: sequence diagram corresponding to fig.3.7

Similarly, the path 10-9-8-7 and the path 12-13-14-15-16 are in parallel with each other between brush B3 and brushes B2/B4. Since, moreover, B1 and B3 are connected to each other externally, all paths are in parallel as shown in fig. 3.9. The machine in our example has 4 poles, 4 brushes, and 4 parallel paths; in general, for lap windings, the number of paths is equal to the number of brushes which, in turn, is equal to the number of poles. Denoting the number of parallel paths by 2a (ie a is the number of pairs of parallel paths), we have

$$2a = 2p \quad \text{(lap).} \tag{3.6}$$

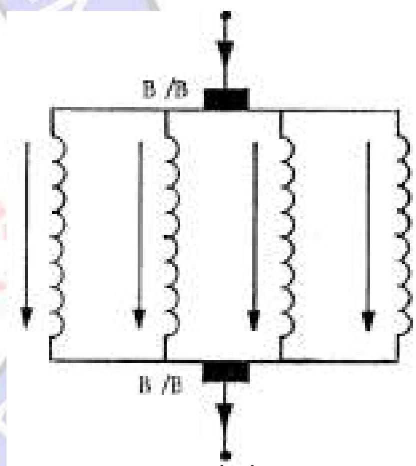


Fig. 3.9 Simple lap winding :parallel paths between brushes.

Of course, because the armature is rotating and the brushes are stationary, the coils forming the paths are changing all the time. Let us assume c1ockwise rotation, and consider coil 10. At the instant shown in figs. 3.7 and3.8, coil 10 is part of the path 10-9-8-7. But in a short while, coil 10 will enter into short-circuit by brush B3, and hence will leave this path. When it comes out from the short-circuit, coil 10 will join the next path, which will be 10-11-12-13-14; in this new path, the current in coil 10 will be reversed :as already explained with reference to fig. 3.7, the arrows in fig. 3.8 are stationary in space.

The above discussion helps explain the function of the commutator and brushes. Consider a coil that is part of a path, with a given direction of current, and a given direction of emf. When the coil reaches the q-axis, it will also be short-circuited by a brush. When the short-circuit ends, the coil will join the next path, with the direction of current reversed; the direction of the emf is also reversed because the coil sides are now under poles of reversed polarity. Thus, the emf and current of the coil are alternating, but the emf and current at the brushes are unidirectional.

3.3.2 Wave Winding

In the wave winding scheme, each coil is connected to a coil that lies approximately two pole pitches away from it on the armature, fig. 3.10. Since the air gap flux density curve is a wave that repeats itself every two pole pitches, fig. 1.24, such coils see almost the same flux density; remember that the same thing was achieved in the lap winding by connecting adjacent coils, section 3.2.1. To connect the coils in wave, the two terminals of each coil are soldered to commutator segments that are separated by approximately two pole pitches on the commutator, fig. 3.10; that is, the commutator pitch y_c is approximately (C/2p)X2 =C/p.

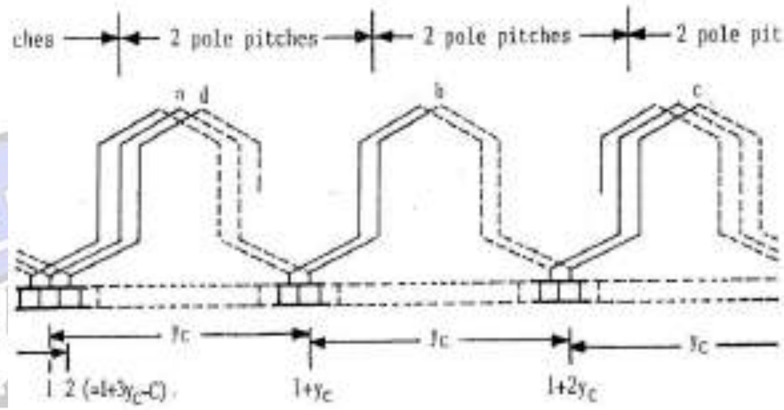


Fig. 3.10 Progressive wave winding for a 6-pole

To find out the exact value of y_c let us follow the coil connections in fjg. 3.10. Coil a start from a segment which can be taken as number 1; the coil then ends on segment number (1 +y_c) according to the definition of the commutator pitch y_c on page 3.4. At segment (1 +y_c) starts coil b, which is approximately 2 pole pitches after coil on the armature; that is, the slot of the first side of coil b is approximately two pole pitches away from the slot of the first side of coil a, and similarly for their second sides. Coil b ends on segment number (l+y_c)+ yc= (l +2y_c).

At segment (1 +2y_c) starts coil c, which is approximately 2 pole pitches after coil b; coil c ends on segment number (l+2y_c) + y_c = (l + 3y_c) at which starts coil d which is approximately 2 pole pitches after coil c. But this means that we have moved 6 pole pitches :2 from a to b, 2 from b to c, and 2 from c to d.

Since the machine in fig. 3.10 has 6 poles, we are back approximately where we started; that is, coil d is next to coil a, and we have gone around the entire armature once. Similarly, we have gone around the entire commutator once, and segment number (l +3y_c) is actually segment number 2, right after segment 1from which we started. Since the number (l +3y_c) is greater than the total number of segments C, we must have

$$1 + 3y_c - c = 2 \rightarrow y_c = \frac{c+1}{3} = \frac{c+1}{p} \qquad (3.7a)$$

This is the exact value of the commutator pitch, and it is slightly different from the approximate value *C/p*. If we attempt to make the commutator pitch exactly *C/p*, then coil c will end on segment 1 instead of 2, and the three coils a, b, and c will form a closed circuit by themselves, which is not what we want :we must go through all the armature coils before closing the circuit, fig. 3.3. The +1 in eqn. 3.7a ensures that, after moving through p coils, we reach the segment after the one we started from. The winding shown in fig. 3.10is progressive; for a -retrogressive winding, we reach the segment before the one we started from. That is, coil c ends on the segment to the left of segment number 1, ie segment number C; thus

$$1 + 3y_c = c \;\rightarrow\; y_c = \frac{c-1}{3} = \frac{c-1}{p} \qquad (3.7b)$$

where the -1 ensures that after moving through p coils we reach the segment before the one we started from. The general expression for the commutator pitch is thus

$$y_c = \frac{c \pm 1}{p} \quad \text{wave} \qquad (3.8)$$

With the positive sign for progressive windings, and the negative sign for retrogressive windings. In both cases, the connection of the end of one coil to tile beginning of a coil approximately 2 pole pitches after it is repeated until the last coil is reached, whose end then closes on the beginning of the first coil to form a closed winding including all the armature coils. Every time we go through p connected coils, we complete one turn around the armature; in lap winding, on the other hand, we have to go through all the armature coils to complete one turn around the armature, figs. 3.6 and 3.7.

As an example, let us try to reconnect the armature of the previous section in wave. The armature slots and coils, the commutator segments, and the poles are all unchanged; the only difference is that the terminals of the coils will now be soldered to different segments. The example machine had 4 poles and 22 armature coils; to connect the coils in wave, the commutator pitch must be

$$y_c = \frac{22 \pm 1}{2} = 11.5 \text{ or } 10.5$$

which is impossible :the commutator pitch has to be an integer because it represents a number of segments, and it is meaningless to have a fraction of a segment. Therefore, this machine, with 2p=4 and C=22. cannot be connected in wave. In general, to be able to connect a winding in wave, the number of poles 2p and the number of coils C must be such that eqn. 3.8 gives an integer answer for y_c; otherwise, the winding cannot be connected in wave. There is no such difficulty with the lap scheme :any number of coils can be connected in lap, since the consecutive coils simply follow each other around the armature.

Let us then change our example a little :instead of 22 coils, we take the number of armature coils to be 21; the number of poles is still 4. The commutator pitch is now

$y_c = \frac{21 \pm 1}{2} = 11 \text{ or } 10$

With a commutator pitch of 11, the winding is progressive, and with a commutator pitch of 10 it is retrogressive; we choose a progressive winding, ie y_c= 11segments. The resulting developed diagram is shown in fig. 3.11. Coil 1 starts from segment 1 and ends on segment 12 (=1 + 11); coil 12 then starts from segment 12 and ends on segment 2 (=12 +11 -21). Thus, by tracing two coils, we have gone once around the armature, and advanced one commutator segment, from segment 1 to segment 2. Coil 2 followed by coil 13 then take us to segment 3, and so on. The last coil which brings us back to segment 1 is coil 11; the sequence of coil connections is shown in fig. 3.12.

Consider next the slots. Assuming one coil side per slot per layer (m =1), the number of slots is equal to the number of coils, S =21 according to eqn. 3.3. The pole pitch is 21/4 =5.25 slots, and the coil span has to be either 5 or 6; the figure is drawn for a short-pitched coil span of 5 slots.It is interesting to note that the winding connections would be unchanged if the armature had only 7 slots, with 6 coil sides per slot (m =3); the coil sides in slots 1, 2, and 3 in the figure would all be placed in a single slot, and the coil sides in slots 4, 5, and 6 would be placed in the next slot, and so on.

As discussed with the lap winding, the coils and commutator segments are in motion, while the poles, q-axes, brushes, as well as the arrows, are stationary. The brushes are placed such that they contact the coils passing through the q-axes. As a coil passes through the q-axis, its current and emf are reversed, which is the principle of commutation.

However, the short-circuiting of the coils by the brushes in the case of wave is different from that in lap. We shall see that a wave winding requires only two brushes; let us therefore assume that brushes B3 and B4 in fig. 3.11are removed. Consider coil 21 :it starts from segment 21, and ends on segment 11, which is free (since B3 is removed). At segment 11 is the start of coil 11, which ends on segment 1. Therefore, brush Bl, which is in contact with segments 21 and 1, short-circuits coils 21 and 11 in series. Similarly, brush B2 short-circuits coils 5 and 16in series. It is noted that all these short-circuited coils, 2l, 11, 5, and 16, are passing through q-axes. Tracing the remaining unshort-circuited coils, they will be found to be in the sequence shown in fig. 3.11, forming two parallel paths between brushes B1 and B2 :the coils in each path are in series; that is the same current flows through them, and their emfs add up, being in the same direction along the path. As each coil passes through the q-axis it leaves one path to join the other, with its current and emf reversed. In general, then, wave windings require only two brushes, and have only two parallel paths, fig. 3.13; thus

$2a = 2$ (wave), (3.9)

Irrespective of the number of poles. This is one of the major differences between wave and lap windings -see eqn. 3.6.

Let us now see what happens when brushes B3 and B4 are put back in place. Brush B3 is connected externally to B1; it is also connected to B1 internally through coils 21 and 10 in parallel. Both coils are in the q-axis with negligible emf, and coil 21 is already short-circuited by B1. Therefore, the only effect of B3 is to remove coil 10 from the active path and add it to the short-circuited coils. Similarly, brushes B2 and B4 are connected externally, as well as internally through the coils 5 and 15 lying in the q-axis. Coil 5 is already short-circuited by brush B2, so that the effect of B4 is to remove coil 15 from the active path and add it to the short-circuited coils.

Therefore, the presence or absence of the additional brushes has little effect on the performance of the armature winding :only q-axis coils, with zero emf, are short-circuited or included with the active paths. The reason for this is that similar brushes (say Bl and B3) are placed two pole pitches apart, and the commutator pitch of the coils is also approximately two pole pitches, so that the brushes are connected internally through coils lying in the q-axis with zero or negligible emf; such coils can be short-circuited, or added to the active path without changing its total series emf. Thus, a wave winding can have 2p brushes, but only two are necessary; the additional brushes are sometimes kept to obtain better current distribution over the commutator. If only two brushes are to be kept, we can choose B1 and B2 as above, or B2 and B3, or B3 and B4, or B4 and BI; that is, one brush from each group.

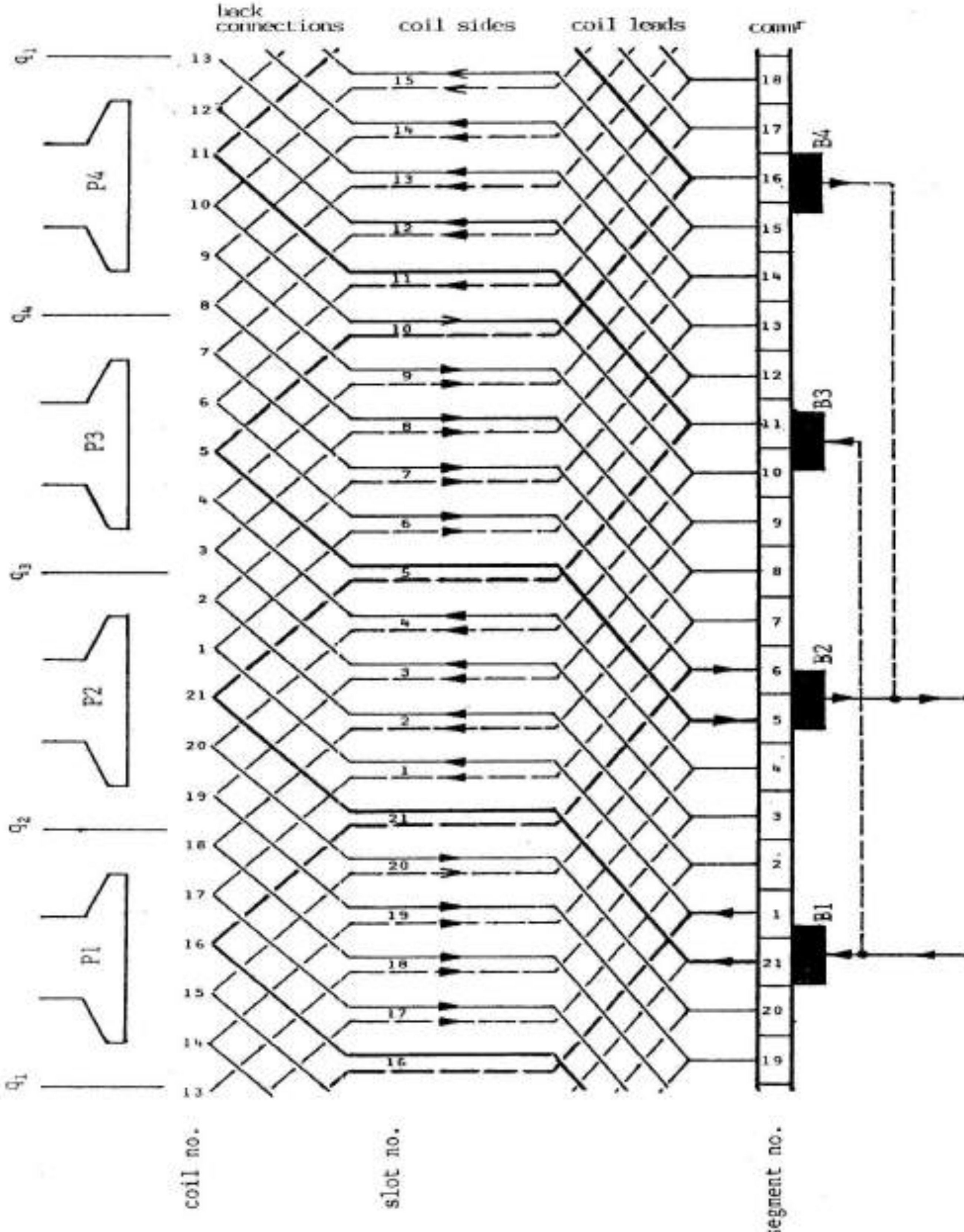


Fig.3.11 Simple wave winding developed diagram for 4 poles and 21 armature coils

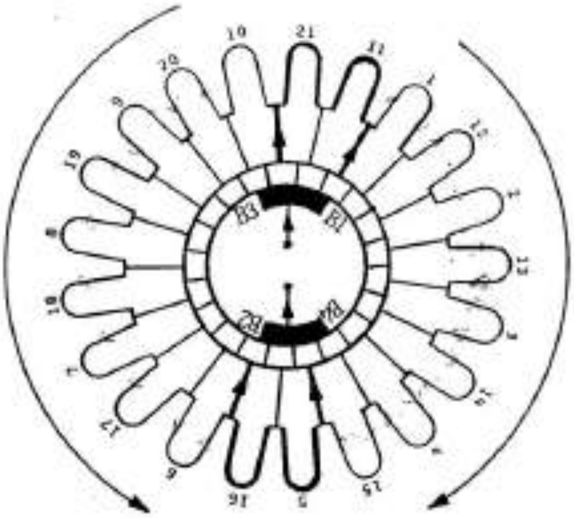


Fig. 3.12 Simple wave winding :sequence diagram corresponding to fig. 3.11

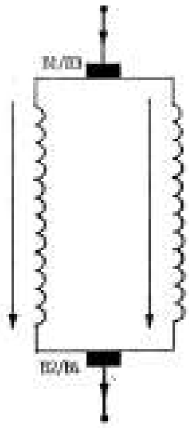


Fig. 3.13 Simple wave winding :parallel paths between brushes

3.3 Armature Calculations

The last section explained how the brushes divide the armature winding into a number of paths, and connect the paths in parallel, figs. 3.9 and 3.13. Each path is composed of a number of coils in series; dividing the total number of armature coils C by the total number of parallel paths 2a, we get

C/2a =average number of coils per path. (3.10)

In chapter 1, section 1.7, it was explained that each coil is represented electrically by its average emf in series with its resistance, fig. 1.28.

Therefore, the electrical circuit of the armature is as shown in fig.3.14a. Summing the coil emfs and resistances in series for each path, the circuit maybe redrawn as in fig. 3.14b. Finally, Thevenin's theorem may be applied to give the simple equivalent circuit of the armature winding shown in fig. 3.14c, where

E_A = equivalent average emf of the armature winding in volts,

And

R_A = equivalent resistance of the armature winding in ohms.

The emf induced in the path is the sum of the series coil emfs in the path; similarly, the path resistance is the sum of the series coil resistances in the path, as there is an average of C/2a coils per path, we have

$$E_{path} = \frac{C}{2a} x E_{coil} \quad \text{and} \quad R_{path} = \frac{C}{2a} x R_{coil}$$

The path emf and resistance are the same for all paths, so that Thevenin's theorem gives

$E_A = E_{path}$ and $\frac{1}{R_A} = \sum \frac{1}{R_{path}} = 2ax \frac{1}{R_{path}}$ → $R_A = R_{path} / 2a$

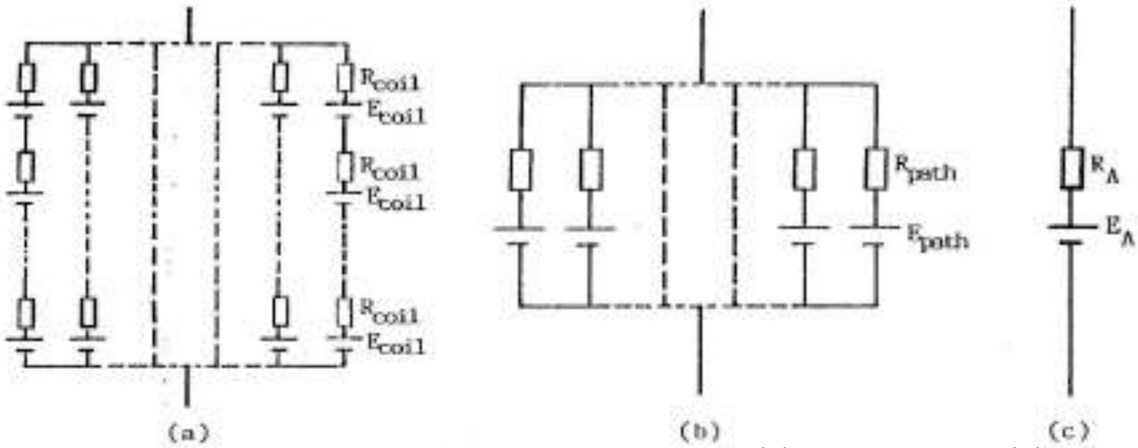


Fig. 3.14 The electrical circuit of the armature winding :(a) detailed circuit; (b) parallel path circuits; (c) Thevenin equivalent.

Using the above resistance equations, together with eqns. 1.29 and 3.1, the armature resistance may be expressed in a number of ways:

$$R_A = \frac{1}{2a} x R_{path} = \frac{1}{2a} \cdot \frac{C}{2a} \cdot R_{coil} = \frac{C}{4a^2} \cdot R_{coil}$$

$$R_A = \frac{C}{4a^2} \cdot NR_{loop} = \frac{NC}{4a^2} \cdot R_{loop} = \frac{Z}{8a^2} \cdot R_{loop} \qquad (3.11)$$

Using, next, the emf equations, together with eqns. 1.13d, 1.15, and 3.1, the armature emf may be expressed in a number of ways:

$$E_A = E_{path} = \frac{C}{2a} x E_{coil}$$

$$= \frac{C}{2a} 4pNn\Phi = \frac{2p\,.2NC}{2a} n\Phi = \frac{pZ}{a} n\Phi = k_e n\Phi$$

$$E_A = \frac{pZ}{a} \cdot \frac{\omega}{2\pi} \Phi = \frac{pZ}{2\pi a} \omega\Phi = k\omega\Phi \qquad (3.12)$$

where $k_e = pZ/a$ and $k = pZ/2\pi a = ke/2\pi$. As already discussed in section 1.6.1, the emf expressions always have a factor representing speed and a factor representing the magnetic field, multiplied by a constant :the emf is induced by the interaction between motion and field.

The voltage between the terminals of the machine and the induced emf in the armature are equal when there is no current flowing in the armature winding; this happens, for example, when the machine operates as an unloaded generator.

However, when current does flow in the armature winding, it will produce a voltage drop in the winding resistance, resulting in a difference between the armature emf and terminal voltage. Current flow through the brushes produces an additional drop composed of two parts. The first is the drop-in brush and commutator segment resistances; this is usually neglected. The second is a potential difference appearing between the brush and commutator surfaces, which are in sliding contact; this is called the 'brush contact drop', and is approximately constant at around (1-2) volts. Now, let

V_A= voltage between machine terminals (armature terminal voltage) in volts,

I_A= total current in the armature winding (armature current) in amperes,

And V_b =brush contact drop in volts.

With the machine operating as a generator, application of Kirchhoff's voltage law to the equivalent circuit as shown in fig. 3.15a gives

$$V_A = E_A - (I_A R_A + V_b) \qquad (3.13)$$

with the machine operating as a motor, fig. 3. 15b, we get

$$V_A = E_A + (I_A R_A + V_b) \qquad (3.14)$$

These are the voltage equations of the dc machine. Under normal operation, the values of induced emf E_A and terminal voltage V_A are close to each other; the difference between them is the voltage drop ($I_A R_A + V_b$), which is usually small compared to E_A and V_A The usual circuit symbol for the armature is shown in fig.3.16.

The total armature current at the brushes I_A is the sum of the path currents; as the 2a parallel paths are identical, they carry identical currents, so that I_A = 2a x I_{path}. Moreover, the path current is simply the coil current

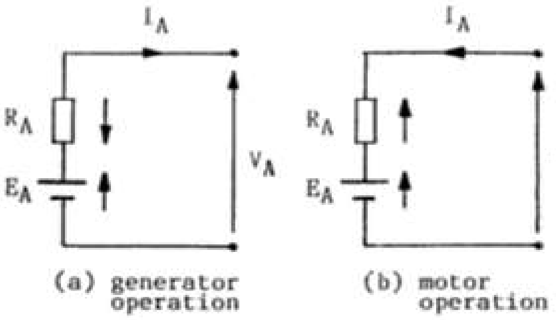


Fig. 3.15 Armature equivalent circuit

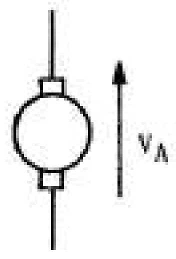


Fig. 3.16 Armature circuit symbol

because the path is just a number of coils in series, fig. 3.14.

Thus

$$I_{coil} = I_{path} = I_A / 2a \tag{3.15}$$

Now, let

T_d = total torque developed electromagnetically by the armature winding (armature developed torque) in newton-meters.

T_d is the sum of the torques developed by all armature coils because the armature winding is arranged so that all coil torques are in the same direction, fig. 1.23, aiding each other. Noting that there is a total of C coils in the armature, and using eqn. 1.24b for the average coil torque, as well as eqns. 3.1 and 3.15, we obtain the following expressions

$$T_d = CXT_{coil} = C\frac{2p}{\pi} \cdot NI_{coil}\Phi = \frac{2pNC}{\pi} \cdot \frac{I_A}{2a} \cdot \Phi$$

$$\therefore\ T_d = \frac{pNC}{\pi a} \cdot I_A\Phi = \frac{Zp}{2\pi a} \cdot I_A\Phi = kI_A\Phi \tag{3.16}$$

where k =(Zp/2πa) as before. As discussed in section 1.6.2, the expression for the developed torque has the current as one factor, and the magnetic field as another, multiplied by a constant the torque is developed by the inter action between current and field.

The power in the emf element of the equivalent circuit is E_AI_A. Using eqns. 3.12 and 3.16, we have

$$E_AI_A = (k\ \omega\ \Phi) \cdot I_A = \omega \cdot (k\ I_A\Phi) = \omega \cdot T_d \tag{3.17}$$

This power, of course, is the conversion power P_c for the whole armature winding; it is given by E_AI_A on the electrical side, and by ωT_d on the mechanical side.

$$P_c = E_A \cdot I_A = \omega \cdot T_d \tag{3.18}$$

The total conversion power in the armature winding is the sum of the conversion powers in the coils; this can be verified using eqns. 3.12 and 3.15:

$$E_AI_A = \left(\frac{C}{2a} \cdot E_{coil}\right)(2a \cdot I_{coil}) = CE_{coil}I_{coil} \tag{3.19}$$

Conversion power and the process of electromechanical power conversion are discussed in section 1.7. The same discussion applies to the armature winding as a whole with, of course, coil quantities (emf, current, resistance, and torque) replaced by the corresponding armature quantities. Multiplying eqn. 3.13 throughout by I_A, we get, for generator operation,

$$V_A I_A = E_A I_A - (I^2_A R_A + I_A V_b) \qquad (3.20)$$

The input power to the electrical circuit is $P_c = E_A I_A$; part of it is lost as heat in the armature winding, $I^2_A R_A$, and in the brush contact, $I_A V_b$, and the rest, $V_A I_A$, is the useful electrical output power obtained from the generator; see also figs. 3.15a and 1.29a. Multiplying now eqn. 3.14 by I_A we get for motor operation

$$V_A I_A = E_A I_A + (I^2_A R_A + I_A V_b) \qquad (3.21)$$

The input power to the electrical circuit is now $V_A I_A$; part of it is lost as heat in the armature winding, $I^2_A R_A$, and in the brush contact, $I_A V_b$, and the rest is the conversion power, $P_c = E_A I_A$, which will appear as, ωT_d, on the mechanical side; see also figs. 3.15b and 1.30a.

3.4 Comparison of Winding Schemes

The equations derived in the previous section apply to both lap and wave windings. The difference between the two types of winding appears through the value of the factor a in the equations :for lap windings, the number of parallel paths 2a is equal to the number of poles 2p, eqn. 3.6; for wave windings, the number of paths 2a is always 2, eqn. 3.9.

First, we note that for a machine having only two poles, there is no difference between lap and wave :in both cases the number of parallel paths is two. In fact, it is meaningless to talk of lap and wave windings when the number of poles is two; there is just one type of winding. Section 3.2.2 explained that in wave winding, each coil is connected to a coil that lies two pole pitches after it on the armature; but if the machine :has only two poles, then moving two pole pitches takes us back to the coil we started from, so that it is connected to the coil next to it, which is precisely the definition of lap winding in section 3.2.1. Also, the commutator pitch for wave is given in eqn. 3.8; for a 2-pole machine, $2p = 2$ and $p = 1$, so that $Y_c = C \pm 1$. Starting from any commutator segment, and moving C segments, we come back to the segment we started from; the ±1 then takes us to the segment just after it, or just before it, which, again, is the commutator pitch for lap.

Clearly, then, it is meaningless to compare lap and wave windings if the number of poles is only two, since in this case they are the same. Therefore, in the following comparisons between lap and wave windings, it is assumed that the number of poles is greater than two, i.e $2p > 2$, and $p > 1$.

Let us suppose that a manufacturer of electrical machines wants to design dc machine having a certain rated voltage and a certain rated current; that is, the values of the terminal voltage V_A and the armature current I_A are given.

The machine can be designed with a lap winding or with a wave winding, and we wish to see the effect of this choice on the design of the individual armature coils. Before starting the analysis, it is recalled from the discussion following eqns. 3.13 and 3.14 that the difference between the terminal voltage V_A and the armature emf E_A is a small voltage drop; neglecting this drop as a first approximation, we can say that the value of E_A is approximately known since the value of V_A is given. Our problem may now be stated as follows:

given the values of the armature emf and current, E_A and I_A, find the values of the coil emf and current, E_{coil} and I_{coil} for the case of lap and the case of wave, and compare the results. From eqn. 3.12, we have:

$$E_{coil} = 2a \cdot E_A / C$$

this gives

$E_{coil,\ lap} = 2p \cdot E_A/C$ and $E_{coil,\ wave} = 2E_A/C$;

thus

$$E_{coil,\ lap} = P \times E_{coil,\ wave}. \qquad (3.22)$$

Using, next, eqn. 3.15, we have

$I_{coil,\ lap} = I_A/2p$ and $I_{coil,\ wave} = I_A/2$;

thus

$$I_{coil,lap} = \frac{1}{p} x I_{coil,wave} \qquad (3.23)$$

Eqns. 3.22 and 3.23 tell us that the lap coils must be designed for higher emf and lower current compared with the wave coils; conversely, the wave coils must be designed for lower emf and higher, current compared to the lap coils. Assuming the two windings have the same number of coils C, the lap winding will have fewer coils in series per path because it has more paths than the wave winding; therefore, each lap coil must have a greater emf to add up to the required armature emf E_A. Moreover, the total armature current I_A is divided over a larger number of paths in the case of lap, so that each path will have a smaller current than the wave paths, which are only two.

The above comparison indicates that in the case of lap winding, each coil will have more turns (higher emf)of thinner wire (lower current); conversely, in the case of wave winding, each coil will have fewer turns (lower emf)of thicker wire (higher current), and this is the main advantage of wave windings overlap windings :with fewer conductors of larger cross-section, a relatively smaller part of the slot cross-section is wasted on insulation; that is, a larger proportion of the slot cross-sectional area is filled with useful copper. The decreased amount of insulation also means that wave-wound coil span be cooled more easily than lap-wound coils. In practice, wave windings are used in all machines rated at high voltage; they are also used in almost all machines of power rating up to 50 KW. Lap windings are used mostly in large machines rated at low voltage and high current.

Next, let us perform an inverse comparison suppose we have an armature whose construction has been completed except for the soldering of coil terminals to commutator segments, and this can be done either in lap or in wave. In this comparison, the coil emf and current are the same for both cases, and we wish to see the effect of the winding scheme on the overall armature emf and current.

Using eqns. 3.12 and 3.15, we can write

$$E_{A,lap} = \frac{C}{2p} E_{coil}\ , \quad E_{A,wave} = \frac{C}{2} E_{coil} \ \rightarrow E_{A,lap} = \frac{1}{p} E_{A,wave}$$

and

$$I_{A,\ lap} = 2pI_{coil}, \qquad I_{A,\ wave} = 2I_{coil}, \rightarrow \quad I_{A,\ lap} = p\ I_{A,\ wave}$$

These relationships indicate that machines with lap windings tend to have low voltage and high current at the terminals. The overall conversion power is the same for both lap and wove wound machines in this comparison.

A lap winding requires 2p brushes, whereas a wave winding requires only two brushes, but can have up to 2p brushes. This is a minor advantage of wave windings :brushes must be accessible for maintenance, and it is easier to arrange for access to 2 brushes on one side of the commutator than for 2pbrushes placed all around the commutator. It is noted in passing that in large machines of high current, the commutator segments can be quite long, and each brush is replaced by a group of brushes, with independent springs, for better contact along the length of the segments.

The magnetic circuit of the dc machine should be symmetric, with the length of the air gap being exactly the same all around the armature. In practice, however, there might be slight variations in air gap length due to imprecision in manufacture or wear of bearings. As a result, the flux density, and hence the induced emf, under one of the poles may be different. This is no problem for wave windings because the coils of each path are distributed all around the armature, so that both paths are equally affected by the fault in the magnetic circuit. In lap windings, however, the fault affects only some of the paths because each path is located under a specific pair of poles; for example, the coils of the upper right path of fig. 3.8 are located under the poles P2 and P3 in fig. 3.7. As a result, the path emf will not be exactly the same for all paths. Unequal emfs in parallel produce circulating current, that is, current that flows from one path to the other without appearing at the machine terminals. Circulating currents heat the coils unnecessarily, and the heating can be quite heavy because the current is limited only by path resistances which are quite small. To reduce this effect, large lap-wound armatures are sometimes equipped with equalizers, which are connections between points in the winding which are two pole pitches apart, that is, points that should beat the same potential; the equalizing connection forces the points to be at the same potential, and hence forces the path emfs to be approximately equal.

Equalizers add to the cost of the machine, and it is an advantage of wave windings not to require them.

The simplicity of the lap winding means that it can be made to fit any number of coils and poles :each coil is connected to the one following it on the armature until the last coil is reached, which then closes on the first coil, section 3.2.1. A wave winding, on the other hand, can be fitted only if the number of coils C and the number of pole pairs p yield an integer value for the commutator pitch in eqn. 3.8. This is no problem at the design stage because suitable values of C and p can always be found. However, it sometimes happens that a manufacturer needs to modify an armature that has already be unconstructed, and the armature is not suitable for the required wave winding.

For example, suppose that the 22-coil winding of fig. 3.7 is to be reconnected in wave; as we saw in section 3.2.2, this cannot be done because the machine has four poles. However, it is possible to form a proper wave winding using only 21 of the coils, with the remaining coil left unconnected. Such an unconnected coil is called a dummy coil; its terminals are not soldered to any commutator segments. Although dummy coils are useless electrically, they are not removed from the armature to maintain mechanical balance.

Example: The following data is given for the armature of a dc machine :conductors/slot/layer =6; commutator segments =146; Coil sides/slot =4; pole arc/pole pitch =0.65; diameter =28 cm; Length =55 cm ; coil span =12 slots ; flux per pole =70 mWb.

A) Find the total number of armature conductors.

B) Find the commutator pitch when the machine is connected in simple wave, and state whether progressive or retrogressive.

C) Find the pole pitch in meters, mechanical degrees, and electrical degrees

Find the average air gap flux density

Solution

Commutator segment=c=146 coils

Coil sides/slot =4=2C/s , so s=2C/4; s=2*146/4=73 slots

conductors/slot/layer =6=NC/S , N=S*6/C =73*6*146=3 turns

A) Z=2NC=2*3*146=876 conductors.

B) Coil span=12= S/2p , 2p=S/12, 2p=73/12 = 6.08 =6 poles

$y_c = \frac{c\pm1}{p}$ wave, $y_c = \frac{146\pm1}{3} = 49$ progressive.

C) Pole pitch=лD/2p =3.14*28/6 =14.65cm

In mechanical degrees= 360/2p =360/6=60 degree

In electrical degrees = 180 degree

In slots =S/2p = 73/6= 12.16 slot.

D) $A_p = \frac{\pi DL}{2p} = \frac{3.14*28*55*10^{-4}}{6} = 805.93 * 10^{-4}\,\text{m}^2$

$B = \frac{\Phi}{A_p} = \frac{70}{805.93*10^{-4}} = 868$ mWb

Example: The table below lists some data on the armature windings of 8 different dc machines. Use the given information to complete the table.

MACHIN NO	3
Number of poles	
Number of slots	12
Number of coils	
Number of conductors	816
Turns /coil	17
Coil sides /slot	
Conductors /slot	
Conductors /slot/layer	
Coil span (in slots)	3
Pole pitch (in slots)	

2p=S/ coil span

2p=12/3=4 poles

C=Z/2N=816/2*17=24 coils

Coil sides /slot=2C/S=2*24/12=4

Conductors/ slot =Z/S =816/12 = 68 cond.

Conductors /slot /layer =NC/S = 17*24/12 =34

Pole pitch (in slots) =S/2p=12/4=3 slots

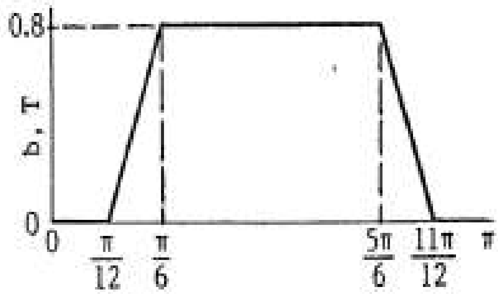
Example: An 8-pole dc generator has 156 slots and 312 commutator segments. The armature coils are connected in simple lap, with each coil made up of 4 turns. The armature rotates at 670 rpm; its length and diameter are 40 cm and 30 cm respectively.

A) How many brushes does the machine have?
B) What is the time for one revolution of the armature?
C) What is the number of conductors per slot per layer?
D) What is the pole pitch? given your answer in meters, electrical degrees, mechanical degrees, and slots.
E) What is the coil span? is it short___, full ___, or long pitched?
F) What will the commutator pitch be if the machine is reconnected in simple wave?

Solution:

A) Number of brushes=2p=8
B) $\frac{t}{1} = \frac{60}{670}$, t=0.0895 sec
C) conductors per slot per layer=NC/S=4*312/156=8
D) pole pitch=л*D/2p=3.14*30/8=11.775 cm
mech. Deg.=360/2p=360/8=45°
elect.deg.=180°
in slot =S/2p=156/8=19.5 slots
E) 19 short, long 20 slots
F) $y_c = \frac{c\pm1}{p} = \frac{312\pm1}{4}$=78.25 or 77.75 so cannot wave.

Example: the flux density distribution over one pole pitch of the machine of Question 8 is as shown in the adjacent figure. (a) estimate the pole arc to pole pitch ratio and the width of pole face. (b)plot the flux density distribution over two consecutive poles; indicate all axes. (c)Also find the average air gap flux density and the flux per pole.



Solution:

(a) pole arc to pole pitch ratio $\frac{y_a}{y_p} = \frac{\frac{5\pi}{6} - \frac{\pi}{6}}{\pi} = 0.667$

width of pole face $y_a = \frac{y_a}{y_p}\, y_p = 0.667 * \frac{\pi D}{2p} = 0.667 * \frac{3.14*30}{8} = 7.85\ cm$

(b) plot

(c) $B_{av} = \frac{1}{2\pi}\int_0^{\pi} b d\theta = \frac{1}{\frac{\pi D}{2p}}\int_0^{\frac{\pi D}{2p}} b dx = \frac{1}{y_{p=\pi}} 0.8\left(\frac{4\pi}{6} + \frac{\pi}{12}\right) = 0.6\, T$

$$\Phi = L\int_0^{\frac{\pi D}{2p}} b dx = L\frac{\pi D}{2p} B = A_p x B = \frac{3.14 * 30 * 40 * 10^{-4}}{8} 0.6 = 28.26\, mWb$$

3.5 Exercises

3.1 the coil span should be approximately equal to a pole pitch. Discuss the effect of making the coil span very different from the pole pitch on (a) the coil emf and (b) the coil torque. Use a diagram like that shown in fig. 1.24, and assume first a short-pitched coil spanning half a pole pitch, then a long-pitched coil spanning one and a half pole pitches.

3.2 What changes must be made in fig. 3.7 if the machine has only 11 slots?

3.3 the q-axis is sometimes called the brush axis; for the same reason, we say that the brushes are located at the q-axis. Discuss the effect of moving the brushes away from the q-axis on

(a) the path emfs;

(b) the developed torque; and

(c) the currents in the short-circuited coils.

3.4 (a) In fig. 3.7, which coils have just completed commutation, and which coils are about to undergo commutation?

(b) Repeat part (a)for fig. 3.11.

3.5 Discuss the effect of removing brush B3 in fig. 3.7.

3.6 : The following data is given for the armature of a dc machine :conductors/slot/layer =6; commutator segments =146; Coil sides/slot=4; pole arc/pole pitch =0.65; diameter =28 cm ; Length =55 cm ; coil span =12 slots ; flux per pole =70 mWb.

D) Find the total number of armature conductors.
E) Find the commutator pitch when the machine is connected in simple wave, and state whether progressive or retrogressive.
F) Find the pole pitch in meters, mechanical degrees, and electrical degrees
G) Find the average air gap flux density

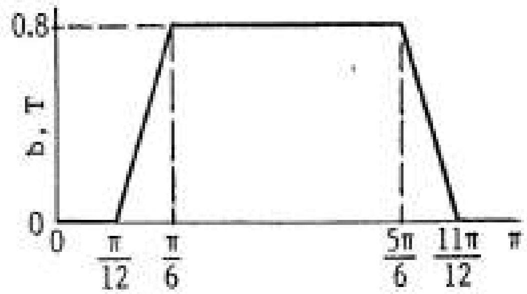
3.7 The table below lists some data on the armature windings of 8 different dc machines. Use the given information to complete the table.

MACHIN NO	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Number of poles	2	2		6	4	6		4
Number of slots	5	9	12	122		97	147	
Number of coils	5				141		441	
Number of conductors			816	244				222
Turns /coil	50	30	17	1	2			
Coil sides /slot		2				4		
Conductors /slot						12		
Conductors /slot/layer					6		6	
Coil span (in slots)			3				18	
Pole pitch (in slots)								

3.8 : An 8-pole dc generator has 156 slots and 312 commutator segments. The armature coils are connected in simple lap, with each coil made up of 4 turns. The armature rotates at 670 rpm; its length and diameter are 40 cm and 30 cm respectively.

G) How many brushes does the machine have?
H) What is the time for one revolution of the armature?
I) What is the number of conductors per slot per layer?
J) What is the pole pitch? given your answer in meters, electrical degrees, mechanical degrees, and slots.
K) What is the coil span? is it short___, full ___, or long pitched?
L) What will the commutator pitch be if the machine is reconnected in simple wave?

3.9: the flux density distribution over one pole pitch of the machine of Question 8 is as shown in the adjacent figure. (a)estimate the pole arc to pole pitch ratio and the width of pole face. (b)plot the flux density distribution over two consecutive poles; indicate all axes. (c)Also find the average air gap flux density and the flux per pole.



3.10 : The armature resistance of the machine of question 8 is 28.8m (ohms. The air -gap filed is as given in question 9. Each coil carries a current of 35 A.

A) Find the resistance per turn.
B) Find the armature current.
C) Find the armature terminal voltage
D) Find the developed torque.
E) Find the conversion power.

3.11 An armature has been completed except for the soldering of coil terminals to the commutator segments, and this can be done in lap or in wave.

Determine the relationship between the armature resistances for the two cases.

3.12 An armature is wound for 2p poles with C coils of N turns each. Let e. i, and r represent the per turn emf, current, and resistance. Write out the expressions for the armature emf, current, resistance, torque, and conversion power for lap connection and for wave connection. Rewrite the expressions in terms of Z, the total number of armature conductors.

3.13 In the table of question 3.6, machines 1-3 are small, while machines 4-8are large. Assume that the resistance per turn is 25 mΩ for small machines, and 1.2 mΩ for large machines.

a. Find the winding pitch and armature resistance for each of the8 machines assuming simple lap windings.

b. Repeat part a assuming simple wave windings.

3. 14 why is the resistance of small machines high, and the resistance of large machines low? why is the resistance of wave windings greater than that of lap windings?

3.15 The windings described in section 3.2 are called simple or simplex windings where a = p, $Y_c = \pm l$ for lap, and a =1, $Y_c = (C\pm 1/\ p)$ for wave. Duplex windings are formed by interleaving two simplex windings on the same armature, and connecting them in parallel; in this case, a =2p, $Y_c = \pm 2$ for lap, and a =2, $Y_c = (C\pm 2)/p$ for wave. Similarly, triplex windings are formed from three simplex windings so that a =3p, $Y_c = \pm 3$ for lap, and a=3, $y_c=(C\pm 3)/p$ for wave. Such windings are called multiplex, and are rarely used.

a. The machine designer may resort to duplex wave windings if the machine being designed has a high current rating. Discuss the reasons.

b. Find the commutator pitch and winding resistance for each of the five large machines of question 3.13 assuming the winding is(i)duplex wave, and (ii)triplex wave.

3.16 A 10-pole simple wave armature has 157 slots, 3 coil sides/slot/layer, and 2 turns/coil. The conductors are made from 3 mm x 20 mm copper strap with a mean length of 160 cm per turn. Find the winding resistance at a hot working temperature of 85 °C.

3.17 A 4-pole simple lap winding is placed in 47 slots with 8 conductors per slot per layer. It is made from no. 12 AWG wire (AWG stands for American Wire Gage; no. 12 AWG has a diameter of 80.8 mils, where 1 inch =1000mils). The winding resistance was measured to be 90 mΩ at 70 °c. Find the mean length per turn.

3.18 A 750 KW dc generator has a terminal voltage of 500 V at rated current. It is being driven at a speed of 450 rpm. The armature resistance is7 mΩ, and the brush contact drop is negligible. Find the armature emf, developed torque, and copper loss.

3.19 Assume that machine number 6 in question 3.6 is wave-wound with each coil having a resistance of 0.6 mΩ. The terminal voltage, current, speed, and brush contact drop are 110 V, 150 A, 900 rpm, and 1 V respectively. Find the flux per pole, torque, conversion power, and copper loss for(a)motor operation, and (b)generator operation.

3.20 A 6-pole dc machine has 46 slots, 184 commutator segments, and 16 conductors per slot. I t is wound in simple wave with 4 mm^2 copper wire. The armature length and diameter are

40 cm and 35 cm respectively. The armature produces an emf of 800 V when it rotates at 500 rpm.

a. Find the number of coil sides per slot, the coil span, and the commutator pitch.

b. Show that the hot armature resistance is approximately 0.6 Ω. NB If the working temperature of the armature winding is not known, a suitable value may be assumed, like 75 ^{o}C or 85 ^{0}C.

c. finds the average air gap flux density.

d. Find the average emf between adjacent commutator segments.

3.21 Assume that machine number 6 in question 3.6 has an average armature emf of220 V, and that the average flux density in the air gap is 0.85 T.

a. at a speed of 800rpm, find the required flux per pole, and hence estimate the value of the product DL assuming (i)simple lap winding, and (ii)simple wave winding. NB the armature diameter x length product DL is a simple measure of machine size.

b. repeat part a at double the speed, ie at 1600 rpm.

3.22 A 10-pole lap-wound generator is rated at 110 V, 600 A, and 750 rpm. The armature has 163 slots with 4coil sides per slot and 2 turns per coi1. The hot winding resistance is 7.2 mΩ, and the brush contact drop is 1.5 V.

a. Find the rated load power, the developed torque, and the flux per pole; also find the per coil and per turn resistances and emfs.

b. The armature coils are reconnected to form a simple wave winding with the speed and flux per pole unchanged. Find the new machine ratings.

c. For both lap and wave connections, find the armature voltage drop in percent of rated terminal voltage, and the copper loss in percent of rated load power.

d. For both lap and wave connections, find the average voltage between adjacent commutator segments.

3.23 Does a dummy coil have an emf induced in it? Should the terminals of a dummy coil be shorted together or left open?

3.24 The armature of a 6-pole dc machine has 116 coils of 2 turns each, with16 conductors in each slot. One of the coils is a dummy coil, and the remaining coils are connected in simple wave. The armature length and diameter are 25 cm. and 20 cm respectively. The average air gap flux density is 0.8 T. The resistance per turn is 2.5 mΩ at 25 0 C.

a. Find the commutator pitch and coil span.

b. Find the hot resistance of the armature winding.

c. If the speed is 900 rpm and the developed torque 138 Nm, find (i)the armature emf, (ii)the total conversion power, and (iii)the current in each coil.

+3.25 A 6-pole machine has 53 slots with 8 conductors per slot. The flux per pole is 50 mWb, and the speed is 420 rpm.

a. In how many ways can coils be formed from the given armature conductors? For each possibility, give. the number of turns per coil, and the total number of commutator segments. (Hint :draw a slot with the conductors inside it, and try to find m.)

b. For each possibility in part a, find the average emf between adjacent commutator segments assuming (1) lap, and (ii)wave connections.

c. In general, the cost of the commutator decreases as the number of segments is reduced. However, the voltage between adjacent segments. should not be allowed to exceed around 20 V to avoid possible damage to the commutator (to be explained in chapter 5). Accordingly, how many segments should the commutator have, and what is the corresponding number of turns per coil for (i)lap, and (ii)wave connections?

d. Verify that each of the possibilities in part a can, in fact, be connected in simple wave.

+ 3.26 A 4-pole lap-wound dc generator is designed to have an average air gap flux density of 0.8 T. The armature winding resistance is 0.3 Ω, and the brush contact drop is 1 V. There is a slight fault in the magnetic circuit which causes the emfs in two of the paths to be 5 %more than the nominal value, and the emfs in the other two paths 5 %less.

a. If the generator supplies a full load of 4.5 kw at 150 V, find the nominal and actual values of path emfs, coil currents, and armature copper loss.

b. Find the nominal and actual values of coil currents and armature copper loss when the generator has no external load.

ANSWERS TO EXERCISE QUESTIONS

2. 10 60 A, 148.4 Nm.

3.7 (1) 500; 2; 100; 50; 2,3; 2(1/2); (2)9;540;60;30;4,5;4(1/2); (3)4;24;4;68;34;3; (4) 122; 2; 2; 1; 20, 21; 20 (1/3); (5) 47; 564; 6; 12; 11, 12; 11(3/4); (6) 194; 1164; 3; 6; 16, 17; 16.167; (7) 8; 1764; 2; 6; 12; 18.375; (8a) 37; 37; 3; 2; 6; 3; 9, 10; 9(1/4); (8b) 111; 111; 1; 2; 2; 1; 27, 28; 271; (8c) 37; 111; 1; 6; 6; 3; 9, 10; 9(1/4).

3.6 a. 876; b. 14.66cm,60degmech, 180 deg elec, 12.167 slots;c. 49 segments, progressive; d. 868 mT.

3.8 a. 8; b. 89.55ms; c.8; d. 11.78cm, 45degmech, 180degelec, 19.5 slots;e. 19 slots short, or 20 slots long; f. X **3.9** a. 0.667, 7.85 cm; c. 0.6 T, 28.3 mWb.

3.10 1.48mΩ, 280A, 780 (-V_b)V, 3145Nm, 220.7KW .**3.11** $R_{A,wave} = p^2 R_{A,lap}$.

3.12 Ze/4p, 2pi, Zr/8p^2, Zei/4πn, Zei/2; Ze/4, 2i, Zr/8, Zei/4πn, Zei/2.

3.13 a. 1.56Ω; 1.69Ω; 0.64Ω; 4.07mΩ; 2l.2mΩ; 19.4mΩ; 16.5mΩ; 8.33mΩ;b. 4,6, 1.56Ω; 8, 10,1.69Ω; X; 41, 36.6mΩ; 70, 71, 84.6mΩ; 65,175mΩ;1l0, 265 mΩ; 18, 19,33.3 mΩ.

3.15 b(i)40, 9.15mΩ; X; 64, 43.7mΩ;X;X; b(ii)X; 69,72, 9.4mΩ; X; 111, 29.4mΩ;17, 20, 3. 70 mΩ.

3.16 135.6 mΩ. **3.17** 61.56 cm .**3.18** 510.5V, 16.25KNm, 15.75KW.

3.19 a. 2.0mWb, 166.5Nm, 15.7KW, 0.8KW; b. 2.2mWb, 183.6Nm, 17.3KW, 0.8KW.

3.20 a. 8; 7 or 8 slots, 61 segments; c. 0.593T; d. 26V.

3.21 a(i) 14.2 mWb, 318.5 cm^2; a(ii)4.7 mWb, 106.2 cm^2; b(i)7.1 mWb, 159.2 cm^2;b(ii)2.4 mWb, 53.1 cm^2.

3.22 a. 66KW, 885Nm, 7.1mWb, 2.2mΩ, 3.55V, 1.1mΩ, 1.78V; b. 556V, 120A, 66.7 KW; c. 5.3 %, *5.3*%, 4.15 %, 4.15 %; d. 3.4 V, 17. 1 V.

3.24 a. 38 seg (retro), 5slots (long); b. 0.17 Ω; c. 433.5 V, 13 KW, 15 A.

3.25 a. (1)l turn, 212 coils, (2)2 turns, 106 coils,(3)4 turns, 53 coils;b(i)4.2 V, 8.4V, 16.8 V; b(ii)12.6 V, 25.2 V, 50.4 V; c(i)53 coils, 4 turns; c(ii)212 coils, 1 turn; d. 71 (prog), 35 (retro), 18 (prog).

3.26 a. nominal :160V, 7.5A, 270W;actual :152 V, 168V, 0.83A, 14.17 A, 483.3W;b. nominal :0A, 0W; actual :6.61 A, 213.3W.

CHAPTER 4
MAIN FIELD

4.1 Introduction

The operation of dc machines is based on the interaction between the armature conductors and the air gap field, resulting in induced emf and developed torque.

Armature windings were explained in the last chapter, and we shall now study the magnetic field. As explained in section 1.5, coils are placed on the stator poles to produce the required field, which is the subject of the present chapter.

Permanent magnet machines, which have no field coils will not be considered here; they are briefly explained in chapter 10 with other special types of de machines.

Now, the purpose of the armature coils is to produce emf and torque. However, when current flows in them, they will attempt to produce a magnetic field, which is called armature reaction. The overall field in the machine will then be the resultant of the main field produced by the field coils and the armature field produced by the armature coils. In this chapter, we shall study the main field by itself; the armature current is assumed to be zero, so that there is no armature reaction. Armature reaction is considered in the next chapter.

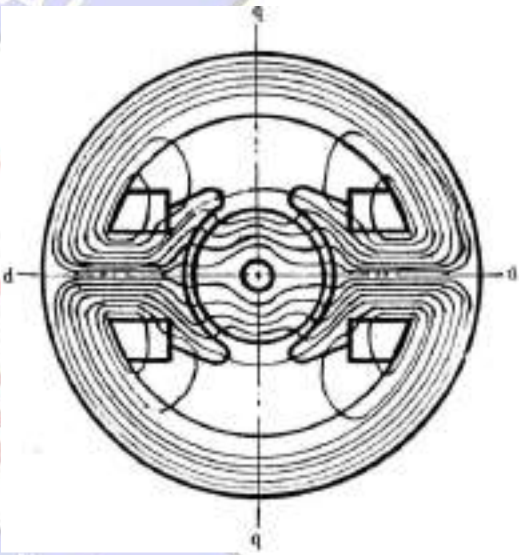


Fig.4.1 Rough sketch of main flux distribution in a 2-pole dc machine.

4. 2 Main Field Distribution

The flux produced by the field coils in a 2-pole machine is distributed approximately as shown in fig. 4.1. Most of the flux in the pole core crosses the air gap to interact with the active conductors at the armature surface; this is the useful flux, Φ in our equations.

However, some flux lines complete their paths without linking the armature conductors; this is the leakage flux, which is not useful because it does not take part in the production of emf and torque. The leakage flux is limited by the high reluctance of the airpaths it follows; usually it does not exceed 10-20 %of the useful flux, which is much less than the amount of leakage shown in fig.4.1

Another approximation in fig. 4.1 is that the flux appears to be uniformly distributed in the air gap between the armature surface and the pole face. This is because the figure assumes the armature surface to be smooth, ignoring the slots and teeth. In fact, the air gap flux has a high density at the teeth, where the air gap is very short and a low density at the slots, where the air gap is much longer, fig.4. 2.

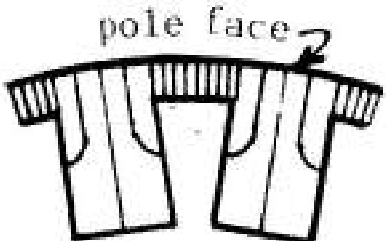


Fig. 4.2 Concentration of air gap flux at teeth.

In a magnetic circuit the flux is determined by the applied magneto motive force, or mmf, and the reluctance in the path of the flux, according to Ohm's law for magnetic circuits:

$$\text{flux} = \frac{\text{mmf}}{\text{reluctance}} \qquad (4.1)$$

The main flux is driven by the mmf coils :if each pole has a coil of N_f turns carrying a current I_f, then the mmf per pole is given by:

$$\text{mmf} = N_f \cdot I_f \quad \text{ampere-turns/pole.} \qquad (4.2)$$

If the machine has two or more coils placed on each pole, then

$$\text{mmf} = \Sigma_j N_j \cdot I_j \text{ Ampere-turns/pole} \qquad (4.3)$$

where the summation is, of course, algebraic, and covers the coils on one pole. This mmf acts on the magnetic circuit in the path of the useful flux. From fig. 4.1, it is seen that the path of the useful flux is composed of the following parts in series :stator, yoke , pole core , pole shoe, air gap , armature teeth and slots , and armature core , at low excitation, when I_f and hence mmf are small the iron parts of the path are in the linear regions of their B-H curves, fig. 2.2; the iron permeability is therefore very high, and its reluctance is very small. Thus, at low excitation, the overall reluctance is approximately equal to the air gap reluctance only. However, as the excitation is increased, iron parts begin to saturate so that their reluctance is no longer negligible compared to the air gap reluctance, and has to be included in the overall reluctance of the magnetic circuit. The highest flux density occurs in the armature teeth, fig. 4.2, followed by the pole core, fig. 4.1, and it is these parts which saturate first.

The mmf of the field coil acts on the air gap along the pole face as shown in fig. 4.3a. The resulting flux density distribution in the air gap is shown in fig. 4.3b where it is assumed that the armature surface is smooth. In fact, the slotting effect causes the air gap flux density distribution to look more like fig. 4. 3c; the ripple moves along the curve as the armature rotates. Whatever the precise shape of the flux density distribution b, the average flux density B and the total flux per pole Φ can obtained by integration on as explained in section 1.5, and it is these values that determined the average emf and torque according to the equations derived in sections 1.6, and 3.3.

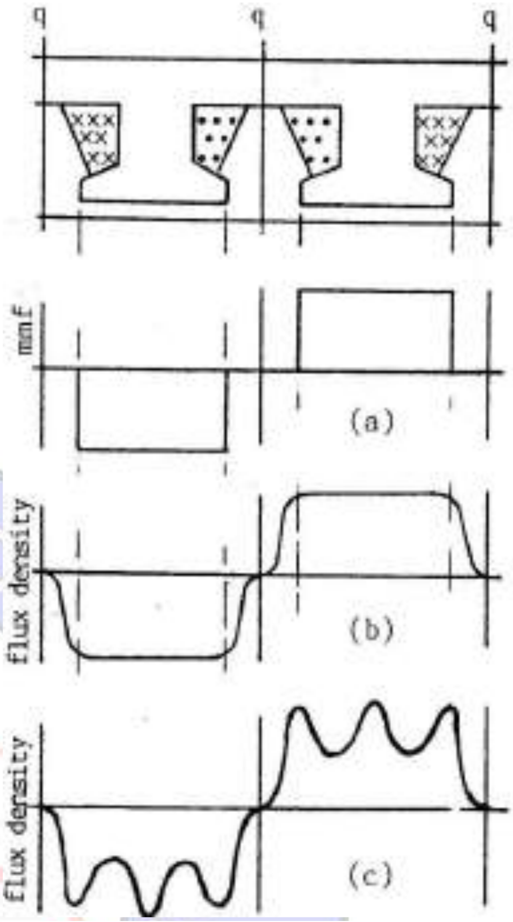


Fig. 4.3 Air gap distributions (a)excitation mmf, (b)flux density assuming smooth armature, and (c)flux density with the slotting effect included.

4.3 Field excitation

The main field is produced by the field coils. The machine may have one set of 2p coils, with one coil placed on each pole; the coils are identical, and are connected in series to form what is called the field winding. Some machines have two or more sets of field coils; the 2p coils of each set are identical to each other (same wire gauge and number of turns), but are generally different from the coil of the other sets. These sets of field coils, or field windings, can be fed with current in a number of different ways. Ina separately-excited machine, fig. 4.4a, the field winding is supplied from a separate source, and there is no connection between the field circuit and the armature circuit.

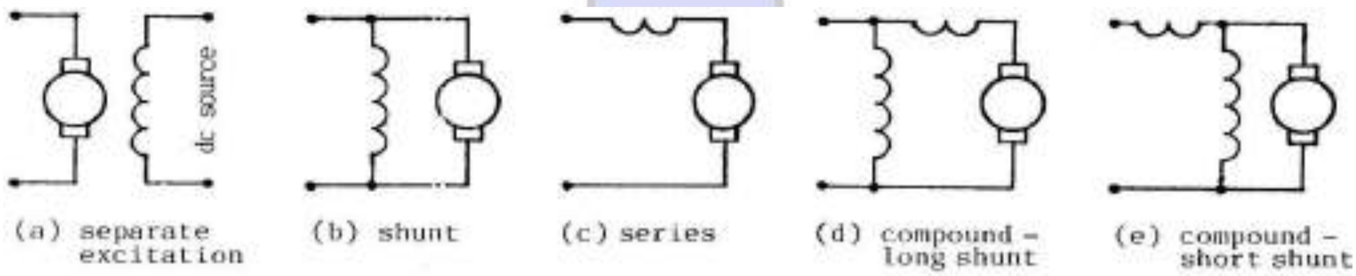


Fig. 4.4 Methods of field excitation in dc machines.

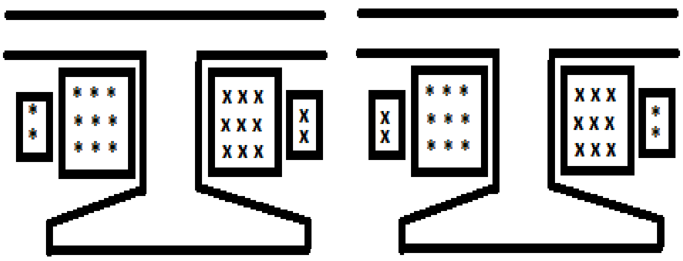
In self-excited machines, on the other hand, the field windings are connected in parallel or in series with the armature, figs. 4.4b-e.

In a shunt machine, the field winding is connected in parallel with the armature, fig. 4.4b. The shunt field coils are made of many turns of thin wire, so that the resistance of the shunt field winding is high, and its current is low; the shunt field current is much smaller than the armature current, usually not exceeding 10 %. The shunt field current can be controlled by means of a variable resistor, or rheostat, connected in series with the shunt field winding. In a series machine, the field winding is connected in series with the armature, fig. 4.4c. The series field coils, are made of few turns of thick wire, so that the resistance, and hence the voltage drop, are low. The series field current can be controlled by means of a diverter, which is a variable resistor connected in parallel with the series field winding.

Compound machines have both shunt and series field windings, that is, two sets of field coils, with two coils on each pole. Compound machines can he connected in long shunt, fig. 4.4d. or in short shunt, fig. 4.4e; there is no major difference between the two connections because, in both cases, the shunt winding voltage is (approximately)equal to the armature voltage, and the series winding current is (approximately)equal to the armature current :the voltage drop in the series winding is small compared to the armature voltage, and the current in the shunt winding is small compared to the armature current. According to eqn. 4.3, the total mmf per pole is given by

$$mmf = N_f I_f \pm N_s I_s \qquad (4.4)$$

where N_f and N_s are the shunt and series turn per pole, and I_f and I_s the shunt and series currents, respectively. In general, the shunt winding mmf ($N_f I_f$)is much greater than the series winding mmf ($N_s I_s$). Usually, the series mmf aids the shunt mmf so that the compounding is cumulative, fig. 4.5a, and the positive sign is used in eqn. 4.4. If, however the series mmf opposes the shunt, the compounding is said to be differential, fig. 4.5b, and the negative sign is used in eqn. 4.4; differential compounding is seldom used.



(a) Cumulative (b) differential

Fig.4.5 Two ways of compounding.

Some special-purpose dc machines have more than two field windings; each winding is fed from a different controlling signal so that the resultant flux per pole is determined by the overall combination of the controlling signals. Such machines are used in automatic control applications.

The main field can also be excited by means of permanent magnets placed on the stator. The absence of field windings in permanent magnet dc machines means that no supply is needed for the field, and operation is economical because there is no copper loss in the field; moreover, they are compact, and hence are widely used as small motors, fig. 2.9, in such applications as toys.

However, the main field of permanent magnet machines is constant, so that they cannot be used in applications requiring variable field. Their main disadvantage is that permanent magnet materials tend to be expensive and difficult to machine; however, there is continuing progress in the technology of permanent magnets, with consequent reduction in production costs. Section 10.1 discusses permanent magnet dc machines in more detail.

4.4 Magnetization curve

The field coils produce the mmf which acts on the reluctance of the magnetic circuit to produce the flux, eqn. 4.1. The magnetization curve is, basically, the relationship between the applied mmf per pole (M_f) and the resulting flux per pole Φ, fig. 4.6; of course, as M_f is increased, Φ increases, but the relationship is not linear because of the iron in the magnetic circuit. The path of the useful flux per pole is composed of the air gap in series with iron parts as discussed in section 4.1; thus eqn. 4.1 takes the form

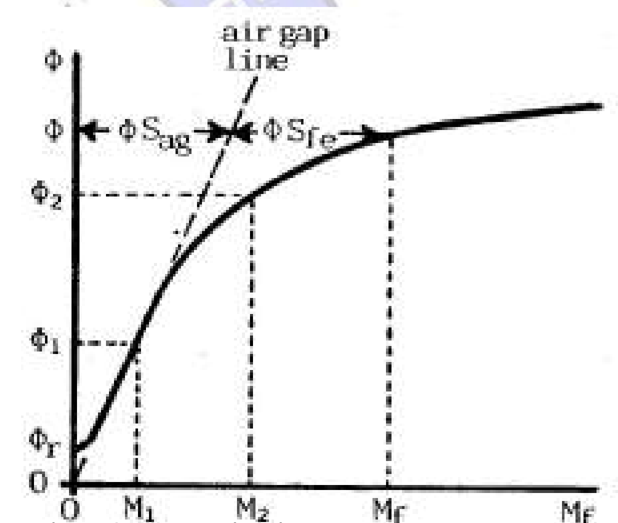


Fig.4.6 Magnetisation curve.

$$\Phi = \frac{M_f}{s_{ag}+s_{fe}} \qquad (4.5)$$

where S_{ag} is the reluctance of the air gap and S_{fe} is the reluctance of the iron parts.

Rearranging eqn. 4.5, we can write

$$M_f = \Phi S_{ag} + \Phi\, S_{fe} \qquad (4.6)$$

this is similar to voltage division in series electrical circuits :ΦS_{ag} is the mmf drop in the air gap, and ΦS_{fe} is the mmf drop in the iron. The air gap permeability is μ_0, so that the air gap reluctance S_{ag} is constant, ie it does not change as the excitation is varied; this gives the air gap line in fig.4.6, $\Phi\, S_{ag}$ increases linearly with Φ. The iron permeability, on the other hand, is not constant, fig. 2.2, so that the value of S_{fe} changes as the excitation is varied. As discussed in section 4.l, at low excitation the iron reluctance and hence mmf drop are negligible compared to the air gap reluctance and mmf drop; thus, at a low excitation like M_1in fig. 4.6, eqn. 4.6 simplifies to

$$M_1 \approx \Phi_1 \cdot S_{ag}; \qquad (4.7)$$

that is, practically all the applied mmf is consumed in crossing the air gap. Thus, at low excitation, the magnetization curve coincides with the air gap line, as shown in fig. 4.6. However, as the excitation is increased, iron parts enter into saturation so that S_{fe} increases, and is no longer negligible; the magnetization curve thus moves away from the air gap line, and at an excitation like M_2 in fig. 4.6, we have

$$M_2 = \Phi_2 S_{ag} + \Phi_2 S_{fe} \qquad (4.8)$$

As indicated on fig. 4.6, the air gap mmf drop ΦS_{ag} is measured between the vertical axis and the air gap line, while the iron mmf drop ΦS_{fe} is measured between the air gap line and the magnetization curve; at low excitation, $\Phi S_{fe}$is negligible so that the curve and line coincide.

The magnetization curve in fig. 4.6 does not start from the origin; with no current in the field coils, Mf =0, there is a small residual flux Φ_r. This is due to hysteresis in the iron, which remains some of its magnetization when the excitation is brought down to zero. To kill the residual flux completely, the field windings must be excited in reverse.

Now, in a mathematical expression Like y =f(x), the term f(x)is a function or formula in terms of x :if we are given a certain value for the variable x, say x_i, we insert this value in f(), and perform the given algebraic operations to obtain the corresponding value of the variable y, namely $y_i=f(x_i)$. If the magnetization curve is given, we can think of it, and use it, just like a function $\Phi = f(M_f)$:given a value of M_f we can find the corresponding value for Φ from the curve; indeed, we can do the reverse: given a value of Φ we can find the corresponding value for M_f from the curve.

Of course, our operations are now graphical, and not algebraic. Suppose, for example, that we need to calculate the developed torque T_d for a given armature current I_A and a given field excitation M_f; to use eqn. 3.16, we need the value of the flux per pole Φ, and this is obtained graphically from M_f using the magnetization curve. Alternatively, the torque may be given, and we need to determine the necessary field excitation; here we use eqn. 3.16 to find Φ and then the magnetization curve to find M_f.

Engineering calculations often use graphical procedures like the one described here in situations where there are no ready algebraic formulas linking the variables, Φ and M_f in our case.

If the machine has one field winding, the mmf per pole M_f is related to the current in the field winding I_f by eqn. 4.2. Since the number of turns per pole N_f is constant, there is direct proportion between M_f and I_f, and the horizontal axis in fig. 4.6 can be scaled in terms of I_f. As an example, letN_f = 500 turns/pole; a point on the horizontal axis marked 2000 A·t/pole can also be marked 4 A of field current; similarly, 3000 A·t/pole becomes 6 A, and so on.

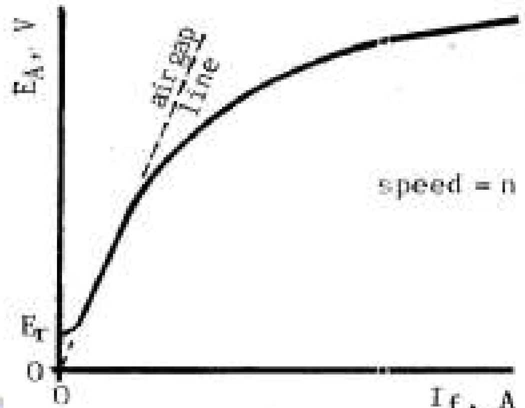


Fig. 4.7 Open circuit characteristic at speed n.

The flux per pole Φ is related to the induced emf E_A by eqn. 3. 12, one form of which is

$$E_A = k_e\, n\, \Phi \quad \rightarrow \quad \Phi = E_A / k_e n$$

if we consider one constant value of speed n, the factor (k_en)will be constant, resulting in direct proportion between Φ and E_A at that speed; the vertical axis in fig. 4.6 can then be scaled in terms of E_A, but the resulting curve is correct for the specified speed -for a different speed, a different scaling factor (k_en)must be used.

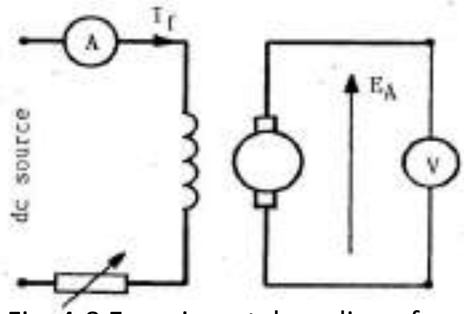


Fig. 4.8 Experimental reading of the OCC. Prime mover rotates armature at constant speed n.

Thus, instead of drawing the magnetization curve as a relationship between the applied excitation M_f and the resulting flux Φ as in fig. 4.6, it can be drawn as a relationship between the exciting field current I_f, and the resulting induced emf E_A (at a specified speed n), as shown in fig. 4.7; in this form, the magnetization curve is called the Open Circuit Characteristic (OCC) of the machine. The OCC can be measured experimentally by the simple test shown in fig. 4.8 :the machine is rotated by means of a prime mover at a constant speed n, usually the rated speed of the machine; the field winding is fed from a separate de source; the exciting field current I_f is varied, by means of n rheostat in the field circuit, or otherwise. The armature circuit is kept open, with no connection to any external load or source; under these open-circuit conditions, a voltmeter connected between the armature terminals as shown will read the induced emf E_A because there is no current, and hence no voltage drops, in the armature circuit -see eqns. 3.13 and 3.14. The emf values are recorded for different values of field current, and the result is plotted as in fig. 4.7. The residual emf E_r is measured when there is no field excitation, ie when the field current I_f is set to zero by disconnecting the field winding from the dc source. Note that in this test the machine is operated as a separately-excited unloaded generator; it does not matter whether the machine is going to be installed as a motor or generator, self-excited or separately-excited, etc. It is also noted that the armature is open-circuited in this test, which is why the resulting characteristic is called the open-circuit characteristic.

Because it can be measured directly, the OCC is the preferred form of the magnetization curve. To scale it back to the original form of fig. 4.6, the horizontal variable I_f is multiplied by N_f, and the vertical variable E_A is divided by ($k_e n$); in practice, however, the constants N_f and k_e are not always known the OCC can still be used, although the original form of fig. 4.6can not be obtained.

Compound machines have two field windings, one in shunt and the other in series, figs. 4.4d and 4.4e. The OCC is obtained with the shunt field winding excited from a separate source as in fig. 4.8, and the series winding left unconnected; this is because the shunt field is the dominant one. Clearly, the horizontal scaling factor N_f represents the shunt turns per pole. But in operation both windings are excited, and the total mmf is their algebraic sum, eqn. 4.4. Given I_f, I_s, N_f, and N_s, the total mmf M_f is computed, and the required point can be located on the horizontal axis of the magnetization curve of fig. 4.6; to locate the corresponding point on the horizontal axis of the OCC of fig. 4.7, M_f must be divided by the scaling factor N_f to obtain an answer in amperes. Dividing M_f in eqn. 4.4 by N_f, we can write

$$\frac{M_f}{N_f} = I_{eq} = I_f \pm \left\{\frac{N_s}{N_f}\right\} I_s \qquad (4.9)$$

where I_{eq} is the required point on the horizontal axis of the OCC; it is an equivalent shunt field current that considers the additional excitation produced by the series field winding. The factor (N_s/N_f) effectively 'refers' the series winding current to the shunt winding :(N_s/N_f)I_s is the current that must flow in the shunt field winding to produce the same mmf produced by I_s flowing in the series field winding.

A given value of field excitation, whether expressed as an mmf M_f or as a field current I_f, produces a specific value of flux per pole Φ, fig. 4.6, but the corresponding emf E_A depends on speed n; this is why the OCC of fig. 4.7 is associated with a particular value of speed :the same value of field current will produce different values of emf at different speeds, although the flux it produces is the same at all speeds. Now suppose the OCC is given at some speed n_1; it is possible to obtain the OCC at any other speed n_2 by simple scaling. Consider a particular value of field current I_o; it produces the mmf M_o and hence the flux Φ_o whose value is determined by the magnetization curve. Now, the emf produced by Φ_o depends on the speed : at n_1, it is

$E_1 = k_e n_1 \Phi_o$, while at n_2 it is $E_2 = k e n_2 \Phi_o$ Dividing, we get

$$\frac{E_2}{E_1} = \frac{n_2}{n_1} \qquad (4.10)$$

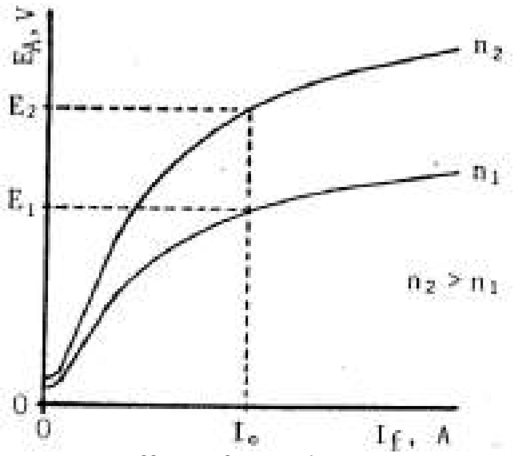


Fig. 4.9 Effect of speed on OCC

That is, at a given field current, the emfs are in the ratio of speeds, fig. 4.9. Thus, if the emf is known atone speed, it can be found at any other speed using

$$E_2 = (n_2/n_1) \cdot E_1 \qquad (4.11)$$

Where E_1 and E_2 are at the same field current. Applying eqn. 4.11 at various values of field current, the OCC at n_2 can be obtained from the OCC at n_l, fig. 4.9.

4.5 Exercises

4.1 Sketch the cross-section of a 6-pole dc machine showing armature, poles, and yoke.

Insert and mark all direct and quadrature axes. Show the main field flux distribution, including leakage.

4.2 The addition of tips to the pole increases the pole arc /pole pitch ratio; this helps improve the air gap flux density distribution and decreases air gap reluctance as explained in section 2.4.1 and fig. 2.5. Why, then, are machines designed with pole arc/pole pitch ratios not exceeding (70 -75) %?

4.3 Fig. 4.2 shows flux lines in the air gap. Complete the figure by extending the lines into the pole shoe and armature teeth. What conclusions can you make regarding flux density, and hence saturation, in teeth?

4.4 Explain how the slotting effect discussed in section 4.1 causes eddy current and hysteresis; losses in pole shoes.

4.5 a; Shunt field windings are designed for low current, ie much less than armature current. Explain why this is desirable, and how it is achieved.

b. Series field windings are designed for low voltage drop, ie much less than armature voltage. Explain why this is desirable, and how it is achieved.

4.6 Draw the connection diagram of a short-shunt compound motor, with resistors to control the currents in the shunt and series field windings.

4.7 Fig. 4.10 shows the terminal box of a dc machine, together with the terminals of a dc source and a resistor. Insert connections for each of the following cases:

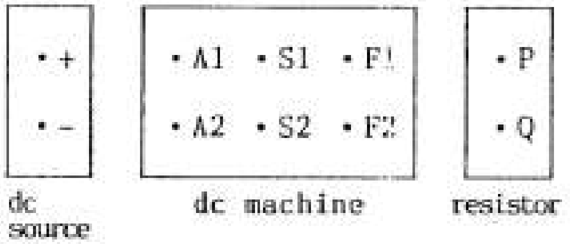


Fig. 4.10 Question 4.7

a. A separately-excited generator loaded by the resistor.

b. A shunt motor fed from the source, with the resistor used for field control.

c. A shunt motor as in part b, but rotating in the opposite direction.

d. A short-shunt compound motor fed from the source, with the resistor used to control the series field current.

4.8 Fig. 4.11 shows a pole excited by five coils, with currents as indicated by the arrows. Write the expression for the total mmf per pole in terms of the currents and turns of the individual coils.

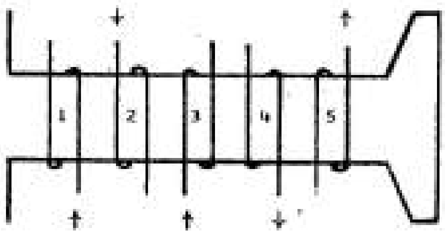


Fig. 4.11 Question 4.8

4.9 why is it not possible to measure the OCC unless the machine is connected as a separately-excited generator at no-load?

4.10 in fig. 4.8, the field current is varied by means of a series rheostat. Show how it can also be varied by means of a voltage divider across the supply.

4.11 The OCC is measured by increasing the field current in steps from zero to some maximum value, and recording the corresponding values of armature emf. If now the current is decreased from maximum to zero, the emf values will be different; why?

4.12 In a compound machine, the shunt field winding has 470 turns/pole, and the series field winding has 13 turns/pole. What current in the shunt field winding produce an mmf equal to that produced by 15 A in the series field winding?

4.13 The armature of a 6-pole dc machine is wave-wound with 46 coils and 3 turns/coil. The field winding has 630 turns/pole. the resistances of the armature and field windings are 75 mΩ and 40Ωrespectively; the brush contact drop is approximately constant at 1.5V. The OCC at 800 rpm is given in table 4.1

Table 4.1 at 800 rpm

I_f (A)	0	0.3	0.5	1.0	1.5	2	2.5	3	3.5	4	4.5	5	6	7	8	9
E_a (V)	8	28	46	94.5	143	175	195.5	211	224	234	243	252	266	277	286	295

a. Plot the OCC at 800 rpm.

b. On the same sheet of part a, plot the OCC at 600 rpm and at 1000 rpm.

c. Plot, on a different sheet, the magnetization curve in the form of fig.4.6, ie flux per pole Φ against mmf per pole M_f.

d. At 800 rpm, estimate the mmf drops in the air gap and in the iron when the emf is (i)100 V,(ii)200 V, and (iii)300 V.

e. what is the residual flux per pole Φ_r?

f. what is the air gap reluctance S_{ag}?

g. Determine the reluctance of the iron parts, S_{fe}, at the 3 emf values of part d.

h. Determine the induced emf when the field current is 5.5 A and the speed is (i)800 rpm, and (ii)1500 rpm.

i. Determine the field current when the emf is 270V and the speed is (i)800rpm, and(ii)1500 rpm.

j. If the field current is 3.8A, at what speed will the emf be(i)200V? (ii) 300V?

k. The machine is operated as a separately-excited motor at 1100 rpm; the armature terminal voltage and current are300 V and 70 A respectively. Find the field current.

l. A series field winding is added to the machine such that when the speed is 500 rpm and the shunt field current is 3 A, the armature emf and current are 150 V and 80 A respectively. Find the number of turns of the series field winding.

Table 4.2 OCC at 1200 rpm

m. The machine is operated as a shunt motor; it draws 80 A from a 220 V source. What is the speed?

4.14 An 8-pole dc machine has 266 armature conductors connected in simple wave. The armature resistance is 0.75Ωand the brush contact drop may be assumed constant at 2.5V. Each field coil has725 turns. The OCC at 1200 rpm is given in table 4.2.

I_f (A)	E_a (V)
0.0	35
0.1	105
0.2	210
0.3	315
0.4	375
0.5	412
0.6	440
0.8	482
1.0	520
1.2	550
1.5	585
2.0	630
2.5	660

a. Plot the OCC at 1200 rpm.

b. What is the flux per pole when the field current is l.3 A?

c. Find the total mmf and the mmf drops in the air gap and the iron when the flux per pole is

(i)14 mWb, and (ii)25 mWb.

d. If the machine runs at l600 rpm with the field excitation set at 0.7 A, what is the emf?

e. If the emf is 380 V at 750 rpm, what is the field current?

f. The machine operates as a generator with a constant field current of 1.4 A, and a constant

terminal voltage 0f 700 v. At what speed will the armature current be zero?

g. The machine operates as a motor at 1500 rpm with a field current of 2 A; it develops 80 Nm. Find the terminal voltage and conversion power.

4.15 The machine of question 4.14 is connected as a compound motor in long shunt, with a series field winding of 20 turns per pole.

a. The motor current is 15 A, and the shunt field current is 1 A. Find the induced emf at 1000 rpm if the compounding is (i)cumulative, and(ii)differential

b. The motor current is 20 A, and the shunt field current is l.5 A; the compounding is cumulative, and the resistance of the series field winding is negligible. If the terminal voltage is 400 V, find the speed, conversion power, developed torque, and armature copper loss.

4.16 The resistances of the armature and field windings of a series machine are 0.1 Ω and 0.05 Ω respectively. The brush contact drop is 1 V. The OCC at 400 rpm is given in table 4.3.

a. The machine is operated as a generator at550 rpm. If the load current is 28 A, find the load power and resistance.

b. The machine is operated as a motor from a battery whose emf and internal resistance are 14 V and 0.07 Ω respectively. Find the speed and conversion power when the motor

current is (i)12A, and (ii)24 A.

Table 4.3OCC at 400 rpm.

I (A)	Ea (V)
5	10.3
10	16.7
15	19.9
20	21.3
25	21.8
30	22.0

ANSWERS TO EXERCISE QUESTIONS

4.12 0.415 A**4.13**d. (i)667A.t, 0; (ii)1333A.t, 324At;(iii)2KAt, 4.1 KAt; e.0.725 mWb;f. 73.6 Kat/Wb; g. 0,17.9 KAt/Wb, 151.3 KAt/Wb; h. (i)259V; 485.6V; i. (i)6.4A; (ii)1.5A;j. (i)695 rpm; (ii)1042 rpm; k. 3.1 A;1.10 turns; m. 658 rpm.

4.14b. 26. 5 mWb; c. (i)203 At, 203 At, 0;(ii)779 At, 367 At, 412 At; d. 616V; e. 1.74A; f. 1461 rpm; g. 802V, 12.57KW**4.15**a. (i)478V; (ii)369V; b. 731 rpm, 7.1 KW, 92.75Nm, 257W**4.16**a. 698 W, 0.89Ω; b. (i)226 rpm, 124W; (ii)142 rpm, 185 W.

CHAPTER 5
ARMTURE REACTION

5.1 Introduction

The last chapter explained the magnetic field produced by the field coils, with no current in the armature. In normal operation, however, there will be current in the armature; its flow in the armature conductors produces an mmf called the armature reaction. The actual flux in the machine is thus produced by the mmf of the field coils and the mmf of the armature acting together. The effect of the armature mmf is to distort the field distribution in the machine; the distortion has a number of adverse effects on the operation of the machine. This chapter explains armature reaction and its effects.

5.2 Distributed armature mmf

When current flows in the armature conductors, it tends to set up a magnetic field. Fig. 5.1 shows the flux distribution in a 2-pole machine due to the currents in the armature conductors acting alone, ie with the field winding unexcited. The conductors form a single 'pseudo-coil', with currents going in at the right and coming out at the left; the axis of the pseudo-coil is therefore the q-axis, and application of the right-hand rule will give the flux direction shown in the figure. Comparing with fig. 4.1, it is seen that the armature field is perpendicular to the main field :whereas the main field acts on the d-axis, the armature field acts on the q-axis. Over a pole pitch corresponding to one pole, the armature flux is directed from the armature to the pole over half the pitch, and from the pole to the armature over the other half pitch; therefore, it tends to aid the main field over half a pole, and oppose it over the next half pole.

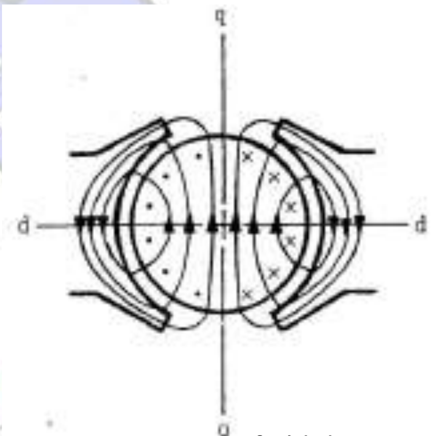


Fig. 5.1 Magnetic field due to armature current acting alone in a 2-pole machine.

Instead of a single pseudo-coil, it is convenient to think of the armature conductors of fig. 5.1 as forming two pseudo-coils :one in the top half of the armature acting to produce a flux directed out of the armature surface, and the other in the bottom half acting to produce a flux directed into the armature.

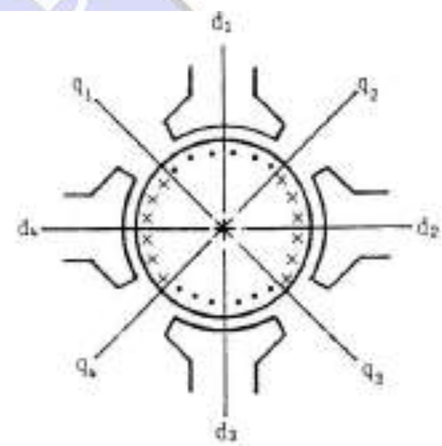


Fig. 5.2 Armature current distribution in a 4-pole machine

Going up to the case of 4-pole machine :fig.5.2. we can distinguish four such pseudo-coils : for example, the conductors enclosed between d_l and $d_2$actto produce a flux directed out of the armature surface in this region; the axis of this pseudo-coil is q_2Similarly, the conductors between d_2 and $d_3$act to produce a flux directed into the armature, and their axis is q_3; and so on for the remaining two pseudo-coils.

The pseudo-coils act on the q-axes, while the main field coils act on the d-axes. The mechanical angle between adjacent q -and d-axes is 45° in the4-pole machine of fig. 5.2, but the electrical angle is 90°. Thus, electrically, the armature field is perpendicular to the main field; this is true for any number of poles since the angle between adjacent q -and d-axes is always 90 electrical degrees.

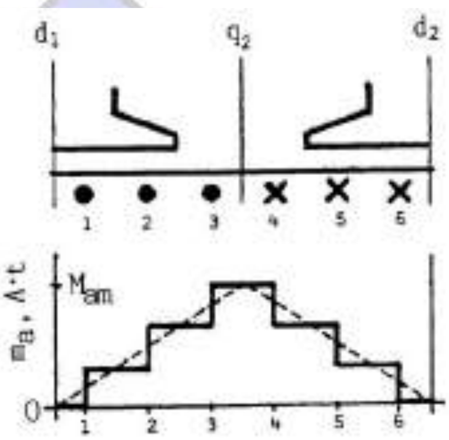


Fig. 5.3 Distributed mmf of armature pseudo-coil

Fig. 5.3 shows a developed diagram of the pseudo-coil between d_1 and d_2 in fig. 5.2, or, equally well, the upper pseudo-coil of fig.5.1. This pseudo-coil attempts to produce a flux directed out of the armature surface, ie upward sin fig. 5.3. The mmf ma represents the action of the conductors of the pseudo-coil in their attempt to produce the flux. The sides at positions 1 and 6 act on the region from 1 to6; the sides at positions 2 and 5 act on the region from 2 to 5; and the sides at 3 and 4act on the region from 3 to 4. By superposing these mmf's, we obtain the overall mmf of the pseudo-coil which has the staircase shape shown:in the regions 1-2 and 5-6, the mmf is produced by the sides at 1 and 6 only; in the regions 2-3and 4-5, the mmf is produced by the coil sides at 1 and 6 as well as the coil sides at 2 and 5; in the region 3-4, the mmf is produced by all the sides, and is therefore maximum, M_{am}. It is convenient to approximate the actual stepped mmf distribution by the triangular distribution shown dotted in fig.5.3. The maximum value M_{am} occurs at the center of the pseudo-coil, ie at the q-axis.

Since M_{am} is produced by all the conductors in the pseudo-coil, it is equal to the total number of turns of the pseudo-coil multiplied by the current in theconductors. The total number of turns is equal to the number of conductors in one side of the pseudo-coil, which is the number of conductors in half a pole pitch, $\frac{1}{2}$(Z/2p); the current in the conductors is the path current I_A/2a. Thus

$$M_{am} = \frac{1}{2}\left(\frac{Z}{2p}\right) \cdot \left(\frac{I_A}{2a}\right) = \frac{Z}{8pa} I_A = \frac{NC}{4pa} I_A \qquad (5.1)$$

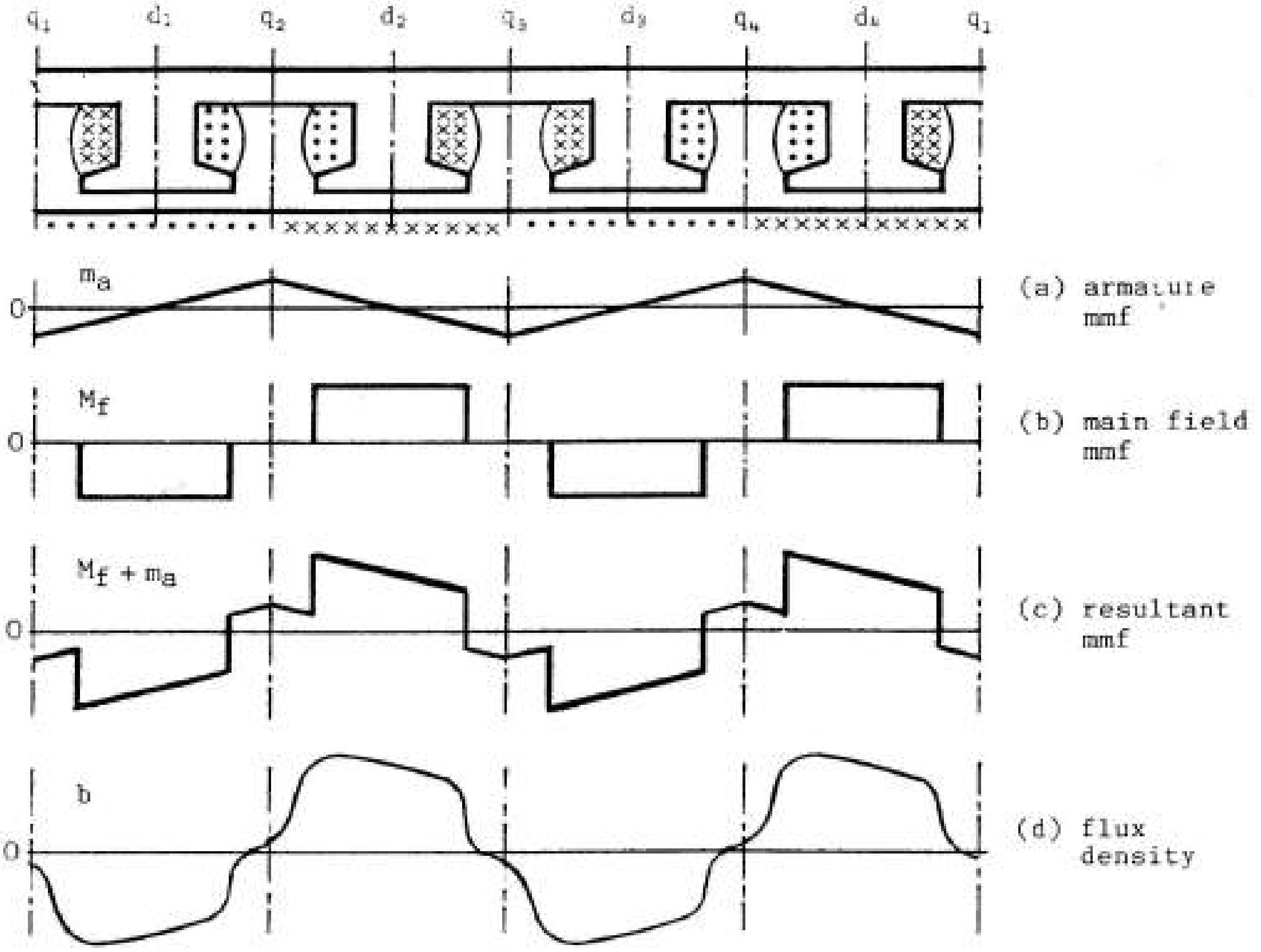


Fig. 5.4 The mrnf distributions acting on the air gap and the resulting air gap flux density distribution.

The stepped distribution approach the triangular distribution as the number of steps is increased, that is, as the number of slots is increased.

Clearly, then, the mmf of an armature pseudo-coil is distributed over the entire pole pitch covered by the pseudo-coil; the statement that "the pseudo-coil acts on the q-axis "simply means that its maximum mmf occurs at the q-axis.

Consecutive pseudo-coils act in opposite directions :one attempts to produce a field directed out of the armature surface :and the next one attempts to produce a field directed into the armature, figs. 5.1 and 5.2. Therefore, if the mmf of one pseudo-coil is taken to be positive, the mmf of the following pseudo-coil must be considered negative. The overall mmf distribution of the armature will thus have the triangular wave-shape shown in fig. 5.4a. Note that the armature mmf is zero at the d-axes.

5.3 Resultant field

In normal operation, there is current in the armature conductors, and the field winding is excited; the respective mmf's, acting on the air gap, are shown in figs. 5.4a and 5.4b. Adding the two mmf distributions point by point gives the resultant mmf distribution shown in fig.5.4c. This resultant mmf acts on the air gap to produce the flux density distribution shown in fig. 5.4d. The flux density wave follows the resultant mmf wave approximately, and not exactly, because the flux is determined not only by the mmf but also by the reluctance, and the air gap reluctance is not constant around the armature periphery :in the region of the d-axis, ie under the pole face, the air gap is short and hence the reluctance is small; in the region of the q-axis, ie between poles, the air gap is long, and hence the reluctance is large. The non-uniform reluctance, and the resulting fringing near the tips of pole shoes, cause the flux density distribution in the air gap to be somewhat different from the mmf distribution producing it. Note, for example, how the wave is rounded at the pole tips due to fringing. Also note that the flux density at the q-axis, although no longer zero, is still very small because of the high reluctance there.

Comparing the resultant mmf and flux density of figs. 5.4c and 5.4d with the main field mmf and flux density of figs. 4.3a and 4.3b, it is seen that armature reaction generally distorts the distributions in the air gap. The distortion is apparent both in the region of the d-axis under the pole, and in the region of the q-axis between poles. Under the pole, the armature mmf aids the main field mmf over half the pole, and opposes it over the other half; the resultant flux density is no longer uniform under the pole, being strengthened at one tip, and weakened at the other tip. Consequently, the instantaneous emf and torque in the armature conductor, which are proportional to b, undergo change as the conductor moves under the pole. Between poles, armature reaction applies an mmf so that the flux density is no longer zero at the q-axis; the point at which b passes through zero is shifted slightly from the q-axis. The presence of a magnetic field at the q-axis means that the emf and torque in a conductor passing through the q-axis are no longer zero; they are still very small because b is very small due to the large air gap, and hence high reluctance, at the q-axis.

Fig. 5.4 includes a developed diagram of the machine showing the sources of the resultant magnetic field, ie the currents in the field coils and in the armature conductors. Application of the right-hand rule indicates that the forces on the armature conductors are towards the right; that is, the torque is in the clockwise direction for the current directions assumed. Therefore, if the motion is also to the right, then the machine is a motor; if, on the other hand, motion is to the left, then the machine is a generator.

In general, then, armature reaction 'pulls' the flux in the direction of rotation in generators, and in the direction opposite to rotation in motors. The distorted flux distribution in a 2-pole machine is shown in fig. 5.5; as in fig.5.4, rotation is clockwise for motor operation, and counterclockwise for generator operation. Compare the resultant flux distribution of fig. 5.5 with the main field distribution of fig. 4.1; note, in particular, that the flux density is no longer uniform around the air gap.

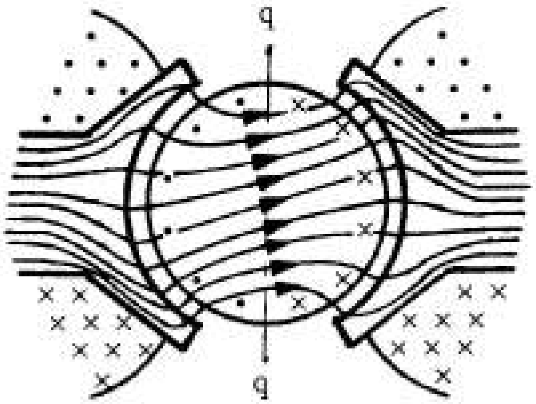


Fig.5.5 Resultant flux distribution in a 2-pole machine, showing distortion due to armature reaction.

5.4 Effects of Armature Reaction

The last section explained how armature reaction distorts the field in the machine. The distortion has three main harmful effects on machine operation; these will now be explained.

5.4.1 Demagnetizing Effect

The armature mmf aids the main field mmf over half the pole, and opposes it over the other half; the increase in mmf in the first half is equal to the decrease in the second half. We might therefore expect the flux per pole Φ to remain unchanged, the increase compensating for the decrease. In fact, however, there is a net decrease in Φ caused by the nonlinearity of Iron: in the region where the field is strengthened, the pole tip and armature teeth are driven deep into saturation, fig. 5.5, so that the increase in flux density there is less than the decrease in flux density in the region where the field is weakened.

Thus, although the increase and decrease in mmf are equal, the resulting increase and decrease in flux density are not, with the increase being less than the decrease. Armature reaction therefore leads to a slight demagnetization of the machine; that is, it reduces the magnetic field in the machine. Usually the reduction is small enough to be neglected, but can become significant for large armature currents, especially under overload or short circuit conditions.

Recalling eqns. 3.12 and 3.16, it is seen that any reduction in the flux per pole Φ results in corresponding reductions in the average armature emf E_A and developed torque T_d.

The OCC is measured with the armature open-circuited so that there is no armature current and hence no armature reaction; let E_o represent the emf under open-circuit conditions. Under normal operating conditions with nonzero armature current, the actual emf E_A may be somewhat less than E_o due to the demagnetizing effect of armature reaction; the difference

$$\Delta E = E_o - E_A \qquad (5.2)$$

thus represents the demagnetizing effect of armature reaction as a reduction-in induced emf. The effective magnetization curve on load, where $I_A \neq 0$, lies below the OCC, and is lower for higher armature currents, fig. 5.6. The curves merge at the air gap line because there is no saturation at low excitation. The difference between the OCC and the effective magnetization curve on load is ΔE, which is a complicated and nonlinear function of both field and armature mmf's, and hence of field and armature currents; in practice, ΔE is much smaller than suggested by fig. 5.6. At a field current I_f and zero armature current, we have

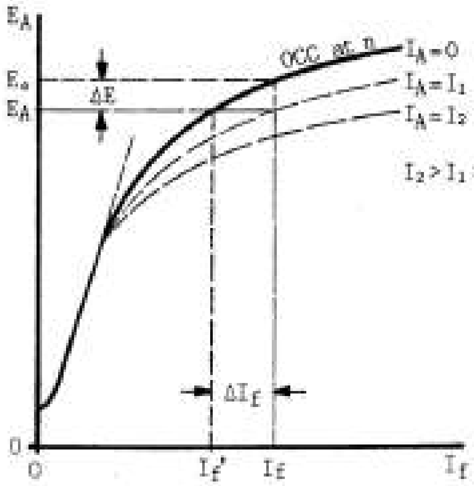


Fig. 5.6 Representation of the demagnetising effect of armature reaction

$$V_A = E_A = E_o \qquad (5.3)$$

At the same field current I_f, but with a nonzero armature current I_A, Kirchhoff's voltage law and eqn. 5.2 yield

$$V_A = E_A \pm (I_A R_A + V_b) = (E_o - \Delta E) \pm (I_A R_A + V_b)$$

$$= E_o \pm (I_A R_A + V_b \mp \Delta E) \qquad (5.4)$$

where, according to fig. 3.15 and eqns. 3.13 and 3.14, the upper sign applies for motor operation, and the lower sign for generator operation. If the demagnetizing effect of armature reaction is negligible, ΔE is dropped from eqn. 5.4, which then simplifies to eqn. 3.13 or 3.14, depending on the mode of operation.

At a given field current I_f and armature current I_A, the point (I_f, EA) represents actual values : If is the actual current flowing in the field winding, and E_A is the actual emf induced in the armature. This point does not lie on the OCC, as can be seen from fig. 5.6; however, it can be associated with two points that do lie on the OCC. The first is the point (I_f, Eo) lying vertically up, and the second is (I_f', E_A) lying horizontally to the left I_f' can be viewed as the effective field current that would give the actual emf E_A on the OCC; the difference

$$\Delta I_f = I_f - I_f' \qquad (5.5)$$

represents the demagnetizing effect of armature reaction as a reduction in field excitation. Thus, the small triangle in fig. 5.6 represents the demagnetization in two ways :the vertical side gives the reduction in emf ΔE, and the horizontal side gives the reduction in effective field current ΔI_f.

5.4.2 Shift of The Magnetic Neutral Axis

The point on the armature surface at which the air gap flux density passes through. zero defines the magnetic neutral axis, or mna. If the main field acts alone, the mna coincides with the q-axis, fig. 4.3. Armature reaction causes the mna to move away from the q-axis, as can be seen in fig. 5.4d. Thus, the flux density at the q-axis is no longer zero, although it is still very small because of the large air gap, and hence high reluctance, at the q-axis. The small field induces a small emf in the coil side passing through the q-axis; this coil side is part of a coil undergoing commutation, ie a coil whose current is being reversed by a brush short circuit -see section 3.2. Thus, the shift of the mna from the q-axis produces a nonzero emf in the coil undergoing commutation.

Another effect of the mna shift is best understood by considering one pole pitch, such as that enclosed between q_1 and q_2 in fig. 5.4d. In this pole pitch, most of the flux is directed downwards, but is reversed in the small region between the mna and q_2; therefore, the conductors passing through this small region will have their emfs and torques opposite to what they should be, resulting in a slight reduction of the overall armature emf and torque. This effect is quite small because the region in question is very small, and the flux density in that region is itself very small.

5.4.3 Flashover

In each pole pitch, armature reaction tends to concentrate the flux at one pole tip, as can be seen in fig. 5.4. Section 5.3.1 explained that this redistribution of the flux can cause a slight reduction in the average emf, and that this reduction is often negligible. Its effect on the instantaneous coil emfs, on the other hand, can be quite significant :a coil whose sides are passing under tips where the field is strengthened will have a high emf induced in it. The coil sides are near the q-axis, which means that the coil is connected to commutator segments that are near the brushes, where the air is highly ionized due to normal sparking between brush and commutator. The high coil voltage is applied between segments, ie it is applied to the ionized air in the region, which may thus break down, resulting in arcing between segments. The heat of the arc can melt holes in the segments. Moreover, the arc may itself cause further ionization and hence further arcing between segments; in severe cases, the arc may extend from brush to brush, which is called flashover.

Flashover is a very serious condition because the heat from the arc can melt the commutator segments, thus causing total machine failure. The machine must therefore be designed to minimize the possibility of flashover. Under normal operating conditions, the coil voltages are unlikely to be high enough to cause flashover. However, under severe overload or short circuit conditions, the distortion caused by armature reaction may be so sharp as to lead to flashover of the commutator.

5.5 Treatment of Armature Reaction

In normal operation, the distortion caused by armature reaction is relatively small, fig. 5.7a. The distortion can become quite sharp if the armature current becomes too high, fig. 5.7b, or if the main field is allowed to become too weak, fig. 5.7c. The adverse effects of armature reaction discussed in the last section place limits on the acceptable operating conditions of the machine; in severe cases, flashover can occur, causing total machine failure, section 5.4.3. Therefore, the design and construction of dc machines must aim at reducing armature reaction and its effects; below we explain some of the treatments used to counter armature reaction.

The machine is designed to have a strong main field; that is, the main field mmf M_f is much greater than the maximum armature mmf M_{am}. The distortion due to armature reaction is then relatively small as in fig. 5.7a; otherwise, the field may be distorted as in fig. S.7c where the shift of the magnetic neutral axis is particularly notable.

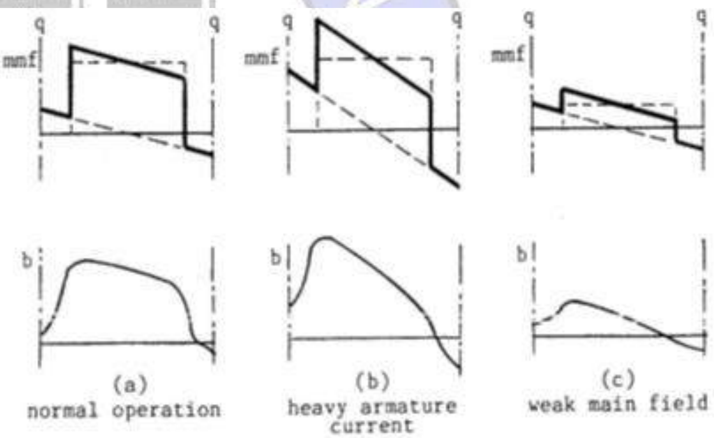


Fig. 5.7 Distortion due to armature reaction under various operating conditions.

Under the pole face, the armature mmf is highest at the pole tips, figs.5.3 and 5.4a. Its effect can therefore be reduced by increasing the reluctance of the magnetic circuit in the region of the pole tip. This may be done by constructing the poles with alternate laminations as in fig.5.8a so that the effective length of the iron in the pole tip is half that in the pole core; with the area seen by the flux in the tip reduced, the reluctance of the air gap under the pole tips is effectively increased. Alternatively, the air gap is made longer under the pole tip by constructing the poles with eccentric faces as in fig. 5.8b, or by chamfering the pole faces as in fig. 5.8c. These methods of increasing the reluctance at the tips are intended to reduce the effect of the armature mmf; of course the increased reluctance also reduces the flux produced by the main field mmf, ie by the main field coils, but the reduction in the main field is much smaller than the reduction in the armature field :In the magnetic circuit seen by the main field, fig. 4.1, the reluctance at the pole tips is in parallel with the reluctance at the d-axis, so that its increase has only a small effect on the overall reluctance seen by the main field; in the magnetic circuit seen by the armature field, fig.5.1, the reluctance at the pole tips is the main series component in the path, so that its increase has a major effect on the overall reluctance seen by the armature field.

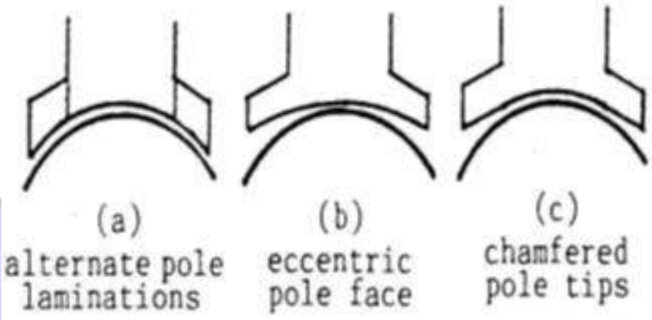


Fig.5.8 Increasing the reluctance at pole tips:((a)each pole lamination has only one tip, and the laminations are stacked alternately; (b)the arc of the pole face is off center with respect to the armature; (c)the pole face is cut straight at the tip.

Section 5.4.3 explained that the ionized air near the brushes breaks down if the voltage between segments becomes too high. Experience indicates that the breakd9wn of air can be avoided if the maximum instantaneous voltage between adjacent segments is not allowed to exceed 30-40 volts; the corresponding average voltage is 20 -30 volts. The machine must therefore be designed in such a way that the average voltage between segments does not exceed 20 -30 volts.

The voltage between segments is directly related to the coil voltage :in a simple lap winding, the voltage between adjacent segments is equal to the coil voltage, and in a simple wave winding, it is equal to p times the coil voltage; in general, then, the voltage between adjacent segments is equal to V_{coil} x p/a.

The design must therefore ensure that E_{coil}xp/a < 20 -30 volts; otherwise, the machine may experience flashover, resulting in total failure.

In some machines, the poles are made with slots in which an additional winding is placed, fig.5.9a; the winding is called the compensating winding or pole-face winding. The currents in the conductors of the compensating winding are opposite to the currents in the armature conductors, fig. 5.9a,so that the corresponding mmf's also oppose, figs. 5.9b and 5.9c; ideally, the two mmf's cancel out under the pole face, leaving only a small mmf acting in the inter polar region, fig. 5.9d. The resultant mmf is then as shown in fig.5.9.e, producing the air gap flux density distribution shown in fig. 5.9f :the distortion under the pole face has been eliminated, as may be verified by comparison with fig. 5.4d; in this way, the compensating winding prevents flashover.

The compensating winding also reduces the flux density at the q-axis and the shift in the magnetic neutral axis; it cannot eliminate them completely because it has no conductors in the inter-poles region, figs. 5.9a and 5.9c.

The compensating winding is connected in series with the armature, fig.5.10, so that the current in its conductors is I_A. The maximum mmf of the compensating winding is

$$M_{cm} = \frac{1}{2} N_c I_A \qquad (5.6)$$

Where N_c is the number of conductors in each pole face. For full compensation as in fig. 5.9, the maximum mmf of the compensating winding must equal the armature mmf at the pole tips, that is

$$M_{cm} = \alpha\, M_{am} \qquad (5.7)$$

Whereα is the pole arc /pole pitch ratio, fig. 1.41. Substituting for M_{am} from eqn.5.1 and for M_{cm} from eqn. 5.6and rearranging, we get

$$N_c = \frac{\propto Z}{4pa} = \frac{\propto NC}{2pa} \qquad (5.8)$$

which gives the number of conductors that must be placed in each pole face for full compensation, ie for complete cancellation of armature reaction under the pole face, fig. 5.9d. The value of N_c obtained from eqn. 5.8 is unlikely to bean integer, and a smaller integer number is used because 60 -70 %compensation is usually sufficient. The connection of the compensating winding in series with the armature fig.5.10 means that compensation occurs at all loads; for example, if the armature current I_A increases, the armature mmf increases, but the compensating winding mmf also increases.

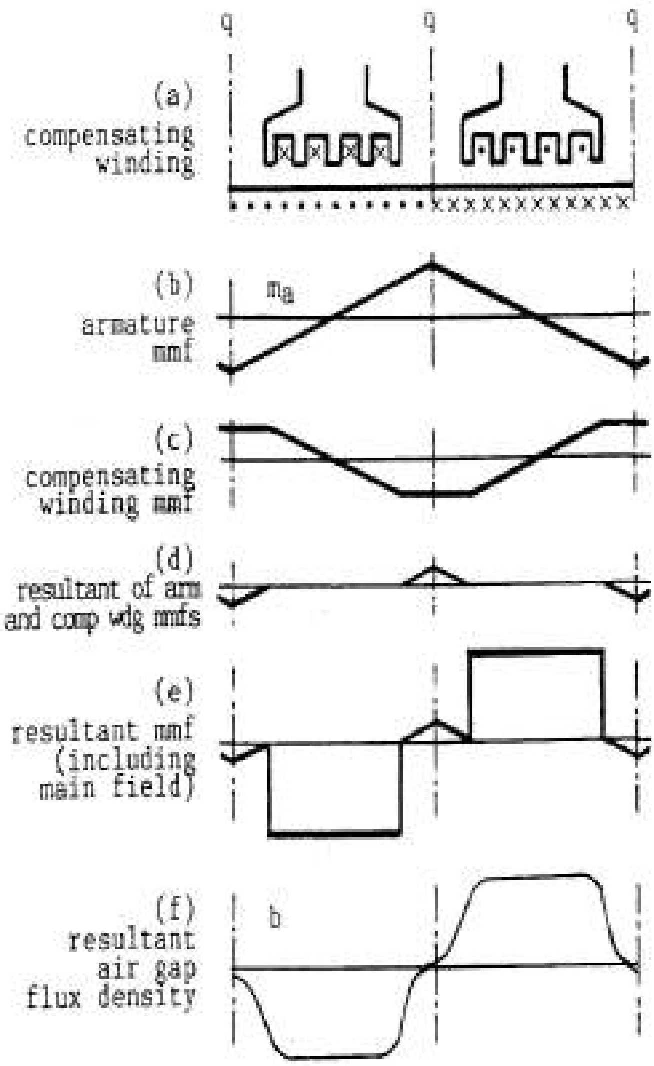


Fig. 5.9 The compensating winding and its effect on the mmf and flux density distributions in the air gap, assuming full compensation.

The compensating winding increases the cost of the machine very much; therefore, it is not used except in very large machines, and in some special-purpose machines, where the high cost may be justified economically.

The orientation of the various windings in fig. 5.10 indicates the axis on which each of them acts. The shunt and series field windings act on the d-axis, which is perpendicular to the brush axis, considered to be vertical through the armature in the figure. The armature and compensating windings act on the q-axis, which coincides with the brush axis.

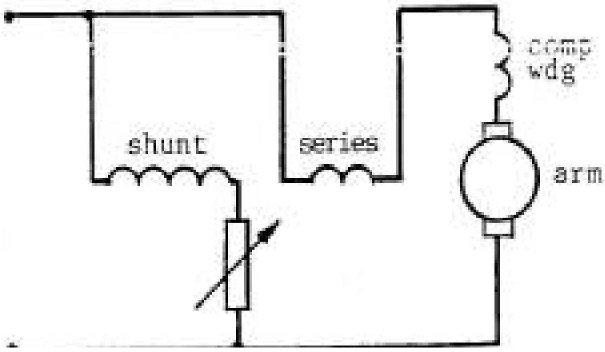


Fig. 5.10 Connection of a compound dc machine, including compensating winding.

5.6 Brush Shift

The brushes are normally located in such positions that they short circuit the coils which are passing through the q-axis; see for example figs. 3.7 and 3.11. The brush axis coincides with the q-axis, where the brush axis is defined as the location of the coil sides of the coils undergoing commutation, ie the coils whose currents are being reversed through brush short circuit. In the armature cross-section, the brush axis is the point at which the conductor currents reverse direction; normally, this is the same as the q-axis. However, if the brushes are moved from their normal positions, the brush axis will be shifted from the q-axis as shown in fig. 5.11 : note in particular that current reversal does not occur at the q-axis. Such brush shift is usually quite small, and results from incorrect positioning during manufacture, or poor brush fit, or normal parts wear over the machine's life, etc. In some small machines, the brushes are shifted intentionally to improve commutation, as will be explained in chapter 6.

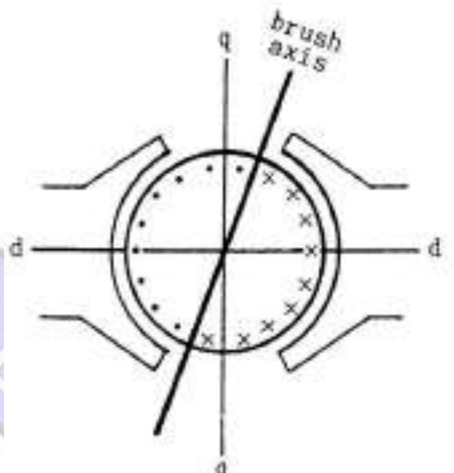


Fig. 5.11 Brush shift, and the resulting distribution of currents in the armature conductors.

Brush shift means that the pseudo-coil formed by the armature conductors does not act exactly on the q-axis, but on the shifted brush axis; that is, the peak of the triangular mmf wave of the armature occurs at the brush axis, which does not coincide with the q-axis, fig. 5.11. The armature mmf may therefore be considered to have two components :a component acting on the q-axis as usual, and a new component acting on the d-axis. Depending on the direction of brush shift relative to the direction of the main field, the d-axis component of armature reaction may aid the main field, or it may oppose it; that is, armature reaction may be magnetizing or demagnetizing. Note that most of the armature mmf still acts on the q-axis; it is 'cross-magnetizing', with a small demagnetizing effect as in section 5.3.1. Also note that brush shift does not alter the distribution of induced emfs in the armature conductor’s reversal of the induced emf occurs at the q-axis and not at the brush axis (more precisely, emf reversal occurs at the magnetic neutral axis).

To check that the brushes are correctly positioned at the q-axis, the machine is connected as a separately-excited generator, and rotated first in one direction, and then in the opposite direction; in both runs, the machine is loaded so that armature current flows, producing armature reaction. The speed, field excitation, and armature current are made the same in both runs. If the brushes are correctly positioned, the magnitude of the terminal voltage turns out to be the same for both directions of rotation. If, however, the brushes are incorrectly positioned, the armature reaction will have ad-axis component which aids the main field for one direction of rotation and opposes it for the other direction; the magnitude of the terminal voltage is then different for the two directions of rotation.

5.7 Exercises

5.1 Redraw fig. 5.4 with the armature current reversed, and compare.

5.2 Using cross-sectional diagrams of a 4-pole machine, show the flux distributions resulting from (i)the main field acting alone, (ii)the armature field acting alone, and (iii)the main and armature fields acting together.

5.3 Using developed diagrams of a 2-pole machine rotating counterclockwise, show that:

a. Armature reaction in a motor (i)increases the flux density under the pole in the direction opposite to rotation, and (ii)shifts the magnetic neutral axis in the direction opposite to rotation.

b. Armature reaction in a generator (i)increases the flux density under the pole in the direction of rotation, and (ii)shifts the magnetic neutral axis in the direction of rotation.

5.4 Using cross-sectional diagrams of a 2-pole machine, show that:

a. In a motor, brush shift in the direction of rotation produces magnetizing armature reaction, while brush shift in the direction opposite to rotation produces demagnetizing armature reaction.

b. In a generator, brush shift in the direction of rotation produces demagnetizing armature reaction, while brush shift in the direction opposite to rotation produces magnetizing armature reaction.

5.5 Draw cross-sectional diagrams, as listed below, for a 2-pole motor rotating clockwise. Show, on each diagram, the q-axis, brush axis, magnetic neutral axis, as well as the currents and emfs in the armature conductors.

a. Motor unloaded; brushes on q-axis.

b. Heavy load on motor, resulting in a 10° shift in the mna; brushes still on q-axis.

c. Motor loaded as in part b; brushes shifted 20° clockwise.

d. Motor loaded as in part b; brushes shifted 20° Counterclockwise.

Study the directions of armature currents and emfs for the various cases, and hence discuss the effects of brush shift. Pay special attention to the region between the brush axis and the mna axis.

5.6 On page 5.5 it is stated that the curves of fig. 5.6 merge at the air gap line because there is no saturation at low excitation; explain in detail.

5.7 Flashover can be considered as a short circuit between brushes; discuss and explain.

5.8 Why is the maximum mmf of the compensating winding less than the maximum mmf of the armature winding? Hence justify eqn. 5.7.

5.9 Explain, with the aid of suitable diagrams, how incorrectly positioned brushes can cause the voltage of a generator to be different for the two directions of rotation.

5.10 Armature reaction can cause demagnetization in two ways; explain.

5.11 A 6-pole dc machine has 95 slots. The armature winding is connected in simple lap with 2 coil sides/slot/layer and 3 turns/coil. The pole face covers 70 %of the pole pitch. The full-load armature current is 600 A.

a. At full load, find the armature mmf at (i)the d-axis, (n)the q-axis, (iii)the pole tip, and (iv)$\pi/3$ electrical radians from the q-axis.

b. How many conductors should be placed in each pole face to compensate for armature reaction? What is the resulting maximum mmf of the compensating winding?

5.12 The commutator of a 6-pole wave-wound machine has 29 segments. Each armature coil has 12 turns and carries 80 A. The machine is connected in long shunt cumulative compound : the shunt field winding has 700 turns per pole and carries 20 A; the series field winding has 25 turns per pole. The pole arc is 45 mechanical degrees. Find (i)the main field mmf per pole, (ii)the maximum mmf of the armature, and (iii)the number of conductors per pole face and maximum mmf of the compensating winding.

5.13 A 2-pole machine is designed to have an armature emf of 600 V. The armature has 35 slots with 16 conductors in each slot. Find the maximum allowable number of turns per coil, and the corresponding number of commutator segments.

5.14 The machine of question 4.14 rotates at 1200 rpm with an armature current of 16 A and a field current of 1.0 A. It is found that armature reaction reduces the induced emf by 38 V.

a. Find the main field mmf and the maximum armature mmf.

b. Find the effective reduction in field current due to armature reaction.

c. Find the effective field mmf.

d. Find the reduction in the flux per pole due to armature reaction.

e. Find the conversion power.

5.15 The machine of question 4.14 rotates at 800 rpm with an armature current of 16 A and a field current of 1.4 A. The armature emf is found to be350 V. Determine the demagnetizing effect of armature reaction as(i)a reduction in induced emf, (n)a reduction in effective field current, and (iii)a reduction in the flux per pole.

5.16 A 4-pole dc machine has a hot armature winding resistance of 0.8Ω; the brush contact drops maybe assumed constant at 1.5 V. The OCC at 1400rpm is given in table 5.1.

Table 5.1 OCC at 1400 rpm.

I_f (A)	Ea (v)
0.0	28
0.1	51
0.2	102
0.3	145
0.4	182
0.6	235
0.8	266
1.0	286
1.5	313

a. The machine is operated as a separately excited generator at 1400 rpm with the field current fixed at 1.2 A. It supplies a 15 Ω load resistor with 5 KW. Determine the demagnetizing effect of armature reaction as(i)a reduction in emf, and (ii)a reduction in effective field current.

b. The machine is operated as a shunt motor from a 220 V source. It is loaded so that the speed is 1000 rpm and the armature current is15 A. Assuming that armature reaction reduces the flux per pole by 5 %, determine the actual and effective field currents; also compute the motor line current and the developed torque.

Table 5.2 OCC at 800 rpm.

I_f (A)	E_A(V)
0	8
0.3	28
0.5	46
1.0	94.5
1.5	143
2	175
2.5	195.5
3.0	211
3.5	224
4.0	234.5
4.5	243.5
5.0	251.5
6.0	265.5
7.0	276.5
8.0	286
9.0	294.5
10.0	302

5.17 The dc machine whose OCC is given on table 5.2 has a wdg resistance of 75 mΩ and a constant brush contact drop of 1.5 v. the mag curves at different armature loadings and constant speed 800 rpm are listed in the adjacent table.

I_f (A)	E_A volts		
	IA=100 A	IA=80 A	IA=60 A
3.5	217	219	221
4.0	225.5	228	230.5
4.5	232.5	235.5	238.5
5.0	239.5	243	246.5
6.0	252	256	260
7.0	262.5	267	271.5
8.0	270.5	275.5	280.5
9.0	277	283	288.5
10.0	283	290	296

(a) Plot these curves, together with the OCC, over the field current range shown here.

(b) Determine E_A, E_0, ΔE, I_f, I_f^*, and ΔI_f when the arm current is 80 A and the field current is 8A and speed is 800 rpm. (c) The machine is operating as a generator at 800 rpm determine the terminal voltage when the field current is7 A and the arm current is 60 A

(d) for motor operation at 1000 rpm with the field current of 7.5A if the armature current is 80 A, find the terminal voltage and the conversion power. (e) if the machine is operating as a 900-rpm generator with terminal voltage 260 V and arm current 100 A, find E_A, E_{AOC}, I_f, I_f', ΔE, and ΔI_f. (f) the machine runs as a motor from a 220 V supply. At a certain load, the speed is 650 rpm and the armature current are 90 A find E_A, E_{AOC}, I_f, I_f', ΔE, and ΔI_f. (g) the machine runs as a motor from a 240 V supply. At a certain load, the field current is 8.4 A and the armature current is 60 A find the speed and developed torque. (h)A series field winding added to the machine such that when the speed is 500 rpm and the shunt field current is 3A, the armature emf and current are 150 V and 80 A respectively. find the number of turns of the series field winding. (I) the machine is operated as a shunt motor; it draws 80 A from a 220 V supply. Estimate the speed.

5.18 Question 5.17h and 5.17i appear to be identical, respectively, to questions 4.131 and 4.13m. why are the solutions and answers different?

Answers

5.11 a. (i)0, (ii)9500At, (iii)6650At, (iv)3167At; b. 22 conductors, 6600 At. **5.12**(i)18 KAt, (ii)4.64 KAt, (iii)43 conductors, 3.44 KAt.**5.13** 4 turns/coil, 70 segments. **5.14** a. 725 At, 133 At; b. 0.2 A; c. 0.8 A; d. 1. 786 mWb; e. 7.7 KW. **5.15**(i)33.3 V, (ii)0.37 A, (iii)2.35 mWb .**5.16** a. (i)9V, (ii)0.145A; b. 1.29A, 1.05 A, 16.3A, 29.6Nm. **5.17**b. 275.5V, 286V, 10.5V, 8A, 6.88A,1.12A; c. 265.5V; d. 346.6V, 27.13KW;e. 269V, 282.9V, 13.9V, 4.98A, 4.25A, 0.73A; f. 211.8V, 221.3V, 9.6V, 6.6A, 5.6A, 1 A; g. 660 rpm, 203 Nm; h. 14 turns; i. 678 rpm

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CHAPTER 6
COMMUTATION

6.1 Introduction

The commutator is a characteristic feature of dc machines. Its purpose is to match the alternating currents and voltages of the armature coils to the direct current and voltage of the brushes as already explained in chapters 1 and 3. However, the commutation process is quite complicated, and gives rise to secondary effects that place limits on the over-all performance of the machine. This chapter explains the commutation process and its main effects.

6.2 The process of commutation

Fig. 6.1 Shows a general arm coil C moving to the right as it rotates with the armature; it is connecter to commutator bars a and b which move with it. (a)When the coil sides are under the poles, the coil is part of a certain armature path and carries a path current I_a:

$I_a = I_A / 2a$

(b)As the coil sides approach the q-axis (or brush axis), there will be an instant t_1 at which the brush contacts bars a and b simultaneously; thus, starts the short circuit of the coil by the brush.

(c)The coil continues to be short-circuited by the brush; it is said to be 'undergoing commutation'.

(d)As coil sides move away from the q-axis, there will be an instant t_2 at which bar b breaks contact with the brush so that the short circuit ends.

(e)Coil sides move under poles, and the coil is now port of a different path; the coil current is I_a again, but in a direction opposite to the original one.

Clearly, then, the coil is short-circuited for an interval T_C

$T_C = t_2 - t_1$

During this interval, the coil current changes from I_a to $-I_a$; i.e. it reverses or 'commutates'. As shown in fig. 6.2, the change in current must follow some time-curve from the point $(t_1,I_a)$to the point $(t_2,-1_a)$. Depending on various conditions that will be explained in later sections, we may have linear commutation (curve 1), over-commutation (curve 2), or under-commutation (curve 3).

To calculate the SC interval (or commutation interval), let u_c denote the speed of the bars; thus

$U_c = 2\pi\, r_c\, n$

Where r_c is the radius at the commutator surface. From fig. 6.3, it is seen that the leading edge of bar a move from x_1 at t_1 to x_2 at t_2; Thus

$$u_c = \frac{x_2 - x_1}{t_2 - t_1} = \frac{w - y_i}{T_c}$$

Therefore

$$T_c = \frac{w - y_i}{u_c} = \frac{1}{2\pi r_c n}\left[\frac{w}{y_o} - \frac{y_i}{y_o}\right] y_o$$

$$= \frac{1}{nC}\left[\frac{w}{y_o} - \frac{y_i}{y_o}\right] \qquad \left(y_o = \frac{2\pi r_c}{C}\right)$$

As expected, the length of the SC interval, T_C, is determined by the speed of rotation n, the relative dimensions of bars and brush, and the number of commutator bars.

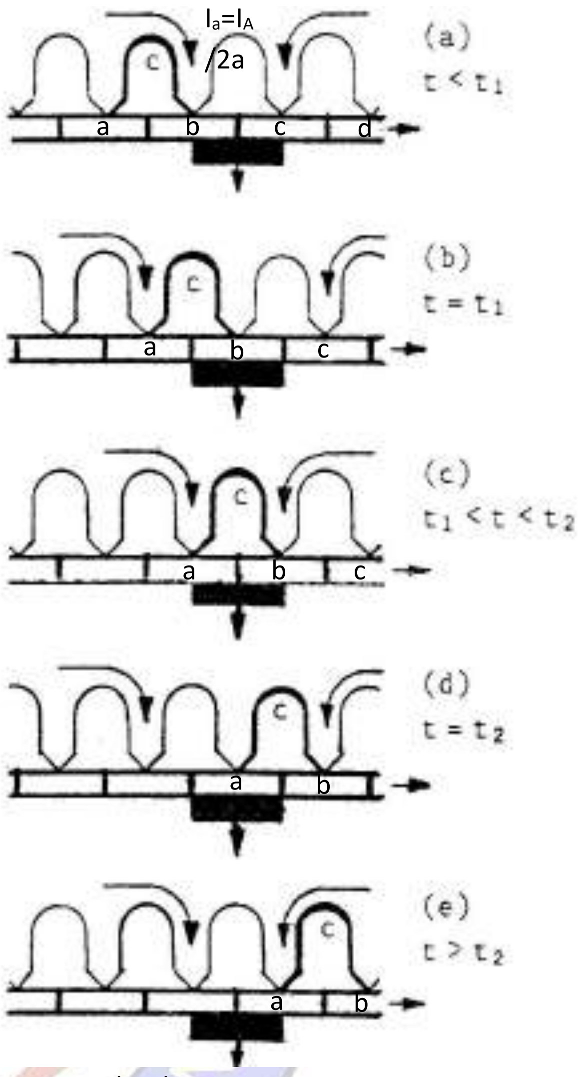

Fig. (6.1)

6.3 Equivalent Circuit of Commutating Coil

During commutation, the coil SC current i_s circulating in a path composed of: the coil itself, risers, bars, contact surfaces, and brush (see fig. 6.4). A

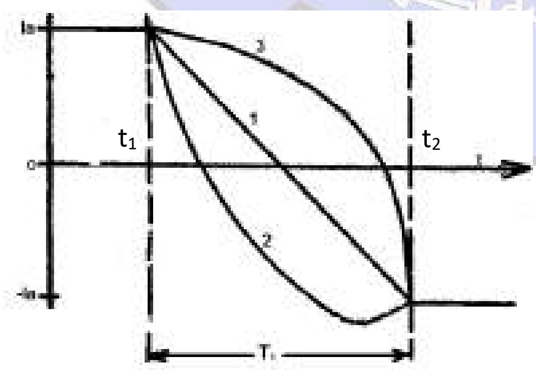

Fig. (6.2)

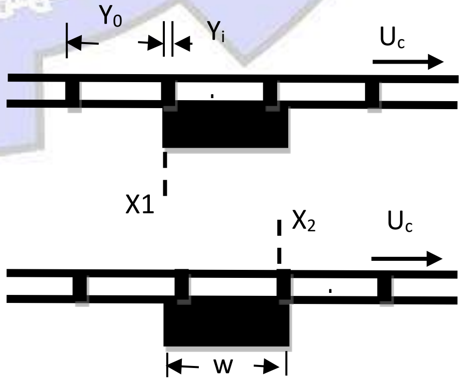

Fig. (6.3)

simplified equivalent circuit is shown in fig. 6.5 with:

R_c = coil resistance;

e_c=rotational emf in coil =2N(bℓu);

L_c = self-inductance of coil;

r_1 =contact resistance between brush and trailing bar;

r_2 = contact resistance between brush and leading bar.

The circuit of fig.6.5 involves the following simplifications:

-The resistance of riser, bar, and brush is negligible w.r.t. contact resistance;

-Mutual inductance with adjacent coils is neglected;

-Brush assumed to short circuit one coil at a time.

Note that lower-case symbols are used for quantities that are time-varying during T_C; these are e_c, i_s, i_1, i_2, r_1, and r_2; indeed, i_s and possibly e_c reverse during T_C. Also note that from KCL

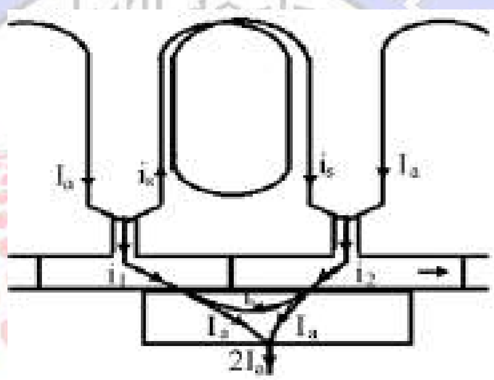

Fig.6.4

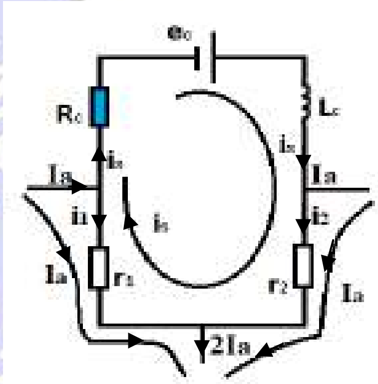

Fig. 6.5

$i_1 = I_a - i_s$ and

$i_2 = I_a + i_s$ So that

$i_1 + I_2 = (I_a - i_s)) + I_a + i_s) = 2I_a$

As expected,

The terminal voltage of the coil v_c is given by

$V_c = e_c - i_sR_c - L_c(di_s/dt)$

The rotational emf e_c is small because field is small around the q-axis (see, for example, fig. 5.5). $L_c(di_s/dt)$ is called the <u>reactance voltage</u>; it is induced by the <u>change</u> in i_s

6.4 Linear commutation (resistance commutation)

In small machines, the coil voltage v_c is smaller than the contact drops $i_1 r_1$ and $i_2 r_2$ we <u>assume</u> that v_c is negligibly small, then the equivalent circuit of fig. 6.5 reduces to that of fig. 6.6. In this case, r_1 and r_2 are in parallel so that by current division:

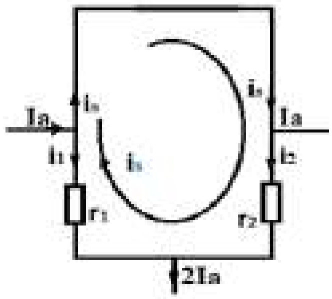

Fig. (6.6)

$$\frac{i_1}{i_2} = \frac{r_2}{r_1}$$

If we further <u>assume</u> that r_1 and r_2 are linear resistances (which in fact they are <u>not</u>), then:

$$\frac{r_1}{r_2} = \frac{A_2}{A_1}$$

Where the contact areas A_1 and A_2 are defined in fig. 6.7. Thus

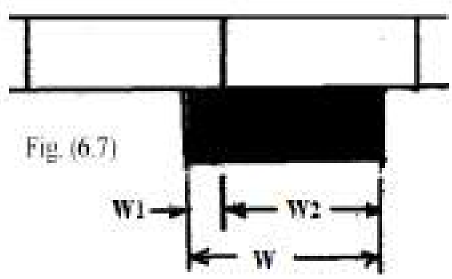

Fig. (6.7)

$A_1 = w_1 \ell_b$, $A_2 = w_2 \ell_b$, $A_b = w \ell_b$

Where ℓ_b = axial length of brush.

$$\frac{i_1}{i_2} = \frac{A_1}{A_2}$$

i.e. the current division between bars is in direct proportion to their respective contact areas. If in the above expression we substitute for i_1 and i_2 in terms of I_a and I_s (see section 6.3), and rearrange, we get:

$$i_s = \frac{A_2 - A_1}{A_b} I_a \text{ (Derive this equation)}$$

As the commutator slides against the brush at constant speed, A_1 increases linearly with time, while A_2 decreases linearly with time. Thus, i_s varies linearly from I_a at t_1 to $-I_a$ at t_2, and we have linear commutation as in curve 1 of fig. 6.2. Linear commutation is also called resistance commutation because the current variation is controlled by the contact resistances r_1 and r_2(see first equation in this section).

Note that we derived linear commutation as an approximation based on two assumptions : negligible v_c and linear contact resistances. These assumptions do not generally hold in practice so that we seldom have linear commutation. Commutation approaches linearity in small machines where these assumptions are approximately true.

6.5 Reactance voltage

Reactance voltage is the voltage induced in the coil due to the time variation of i_s; it appears across L_c in the equivalent circuit of fig. 6.5, and is equal to $L_c(di_s/dt)$. Reactance voltage has a great effect on the commutation process, so that linear commutation. The role of reactance voltage in the commutation process may be described qualitatively as follows:

The reactance voltage is induced by the change of coil current from 1_a to -1_a. According to Lenz's law, the reactance voltage will be induced in such a way as to oppose what is causing it, i.e. it opposes the change in current. Therefore, the reactance voltage retards or delays the change in current.

Due to reactance voltage, then, the current tends to follow a curve above that of linear commutation, for example curve 3 in fig. 6.2; the greater the coil inductance L_c, the higher the curve.

If current reversal is not complete (i.e. current has not reached-I_a) when bar b breaks contact with the brush at t_2, the curve will be as shown in fig. 6.8. This results in sparking which is explained as follows:

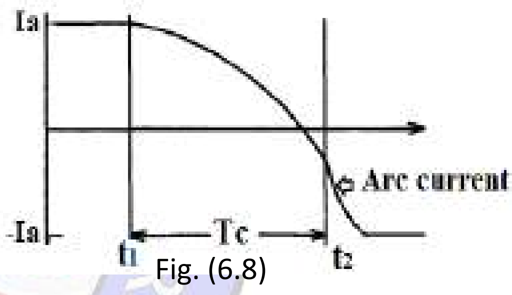

Fig. (6.8)

At t_2 the coil current attempts to jump to $-I_a$ almost instantaneously. This results in very high reactance voltage (why?), which causes breakdown in the air. The arc provides a path between brush and bar b through which current flows to complete its reversal to $-I_a$.

Sparking is harmful because it causes heating and hence wears of both brush and commutator bars. It becomes more severe as load increases (as the armature current I_a increases, so does the path current I_a).

6.6 Treatment of Sparking:

In some small machines, the resistive contact drop is much greater than the reactance voltage so that sparking is limited by the effect of resistance commutation, i.e. commutation approaches the linear case.

In larger machines, some additional means must be found to limit sparking, i.e. to counter the effect of reactance voltage which is the prime cause of sparking as explained above. Modern machines use interpoles, while older machines (and some small machines) use brush shift; these two methods are explained in sections 6.6 and 6.7.

6.7 Interpoles (Commutating Poles)

Nearly all integrating machines have interpoles. Interpoles are narrow pole with air-gap placed between main poles, as in fig. 6.9. There coils are connected in series with the armature so that the interpole field is proportional to armature current I_A (the large air gap prevents saturation in the iron). The interpole field acts on commutating coils at the q-axis. The interpole mmf M_i is given by

$$M_i = N_i I_A$$

Where N_i is the number of turns in each interpole coil. The number of turns N_i is chosen to make the interpole mmf some 25 %greater than M_{am}, the cross-magnetizing armature mmf at the q-axis (see section 5.1); thus M_i=1.25 M_{am} so that N_i =1.25(NC/4pa)

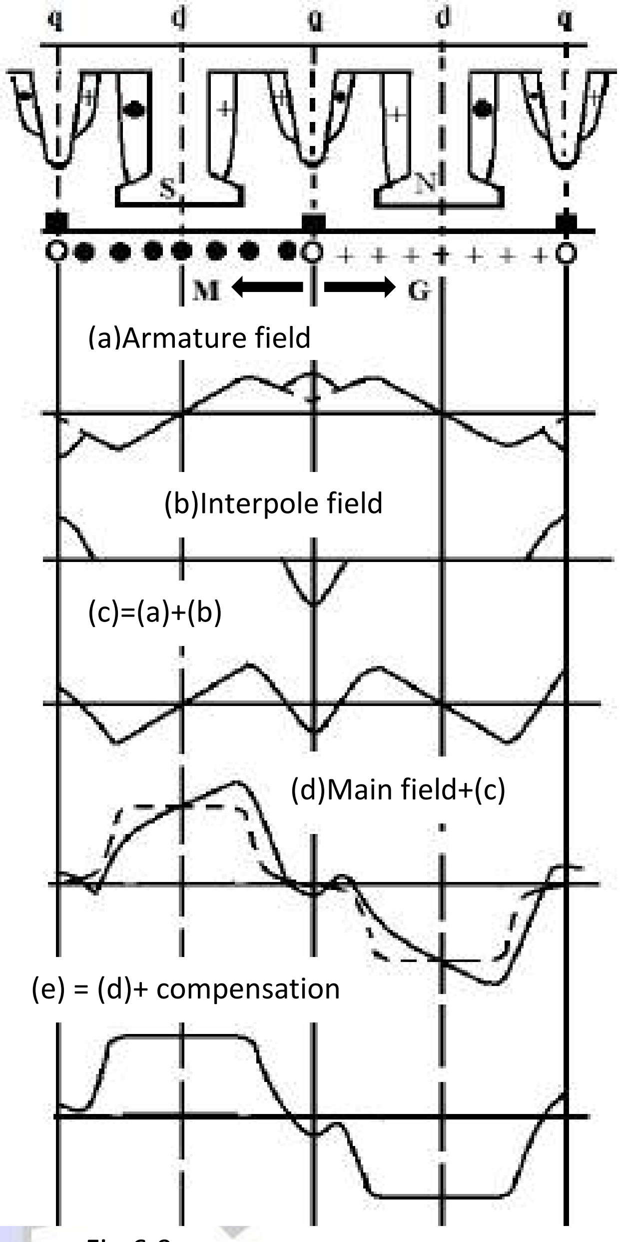

Fig.6.9

In this way, M_i is made to serve two purposes :(1) It fully neutralizes the armature reaction field, and (2) the additional 25 %neutralizes the commutating coil flux (which induces the reactance voltage). This is clear in figs. 6.9 a, b, and c :the interpole field not only reduces the q-axis field to zero, but drives additional flux in the negative direction to neutralize reactance voltage. Fig. 6.10 d and e show the resultant field in interpole machines, without and with compensating windings; compare them with figs. 5.5 and .6.9

(NB interpoles treat arm reaction in the q-axis, while compensating wdgs treat arm reaction under the poles).

As the machine is loaded, the armature current I_A increases so that armature reaction and reactance voltage increase; but the interpole field is also proportional to I_A, and will increase automatically to neutralize armature reaction (in the q-axis) and reactance voltage. Interpoles will continue to do their job properly for either mode of operation, motor or generator, and for either direction of rotation, forward or reverse.

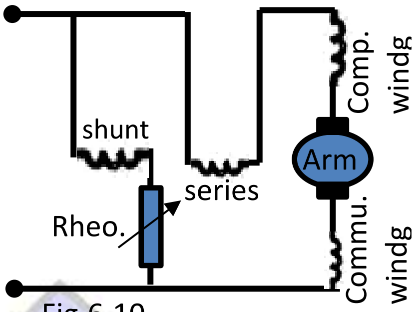

Fig.6.10

Fig.6.10 shows the general connection of a dc machine. Not all windings shown are present in all machines. Interpole or commutating wdgs are found on integral horsepower machines (rated power greater than one hp); compensating wdgs are found on large machines and on some special machines; many machines have only one main field wdg, shunt or series; compound machines have both. The terminals of main field wdgs (shunt and series) are usually brought out to the terminal box to allow user manipulation; the terminals of compensating and commutating wdgs are not brought out to the terminal box so that they are permanently connected in series with the armature.

6.8 Brush Shift

A second method for improving commutation to limit sparking is to shift the brushes from the q-axis. The principle is as follows:

Recall figs. 5.4 and 5.5 which show how the magnetic neutral axis (mna :b=0) moves away from the q-axis due to armature reaction. If now the brushes arc shifted in the same direction, they will be in a region where the armature field opposes the main field. At some location the two fields cancel out; placing the brushes at this location eliminates the rotational emf e_c (see fig. 6.5). This is not enough because there still is the reactance Voltage. To neutralize reactance voltage, the brushes are shifted a little further in some direction; the sides of commutating coil will then be subjected to a small (but nonzero) field that opposes the coil flux which induces the reactance voltage. If the opposing fluxes can be made equal, the reactance voltage is eliminated.

As a method for improving commutation, brush shift is not as good as interpoles because it has the following disadvantages:

1. As the load on the machine changes, the arm current 1_A changes so that AR and the mna shift also change. For correct operation, the brush shift must be changed accordingly, which is impractical. In practice, the brushes are placed in a position that gives minimum sparking at rated load, so that there may be considerable sparking at other loads (eg no-load!)

2. From fig. 5.4, it is seen that the mna shifts in the direction of rotation for generator operation, and in the direction opposite to rotation for motor operation. Thus brush shift (which is in the same direction as the mna shift) cannot be used with motors intended to run in both directions, or with machines intended for variable mode of operation :if brush shift is correct for one case, it is incorrect for the other.

3. Brush shift :causes demagnetizing armature reaction

Because of these disadvantages, brush shift is used only in small fractional horsepower machines (rated power less than one hp) where it is not economical to use interpoles. Brush shift was also used in old machines before the invention of interpoles.

6.9 Exercises

6.1 The armature of a dc machine has 268 coils and rotates at 900 rpm. Each brush covers four commutator segments. Find the time for one revolution of the armature, and the interval during which a brush short-circuits a coil.

6.2 The armature of an 8-pole machine has 375 coils connected in simple lap. Each segments of the commutator are 9.5 mm wide, and the mica insulation between segments is 1.3 mm wide. The brush width is 38mm.

a. Find the diameter of the commutator.

b. Find the commutation interval.

c. for what fraction of a revolution is a coil short-circuited by brushes? For what fraction is it active?

d. on average, how many coils are short-circuited by brushes at any given instant?

6.3 A 4-pole lap-wound dc machine has 65 armature coils; the inductance of each coil is 0.02 mH. Each brush covers two commutator segments; the width of insulation between adjacent segments is 1/7 the width of the copper. The armature current is 80 A, and the speed is 1150 rpm. Assuming linear commutation, find the reactance voltage in coils under-going commutation.

6.4 An 8-pole dc generator has 156 slots and 312 commutator segments. The armature coils are connected in simple lap, with each coil made up of four turns. The armature rotates at 670 rpm; its length and diameter are 40 cm and 30 cm respectively. Each brush covers 3.5 segments.

a) Find the time interval during which the coil is short –circuited by brush.

b) If the machine is fitted with interpoles, find the number of turns of each interpole coil.

6.5 A shunt motor draws 75 A at 220 V and 650 rpm. The field current is 3 A. the armature has 41 slots, and the commutator has 123 segments. Each armature coil has 4 turns and carries a current of 12 A. if the machine has interpoles, how many turns should each interpole coil have?

6.6 A 4- pole dc motor has interpoles, and rotates counterclockwise.

a) Draw a developed diagram indicating the currents in the armature, main field coils, and interpole coils. Also sketch the various mmf distributions, and the resultant air gap flux distribution.

b) draw the machine cross-section and sketch the flux distribution.

6.7 show by means of suitable diagrams that when the direction of rotation of a motor is reversed, the interpoles continue to do their job correctly.

6.8 in which mode of operation of a dc machine does the interpole have the polarity of the main pole that comes after it ?

6.9 justify the following statement: if the interpole winding is left unexcited, commutation will be much worse than if the machine did not have interpoles.

6.10 draw a developed diagram covering two poles of a large machine that has interpoles as well as compensation windings. Indicate all currents, and sketch their mmf distributions. Also sketch the resultant flux density distribution in the air gap.

6.11 A 6-pole dc machine has 95 slots, the armature winding is connected in simple wave with 2 coil sides per slot per layer, and 3 turns per coil. The pole arc covers 70% of the pole pitch.

a) Find the number of turns in each interpole coil if the machine has interpoles but no compensation winding.

b) Find the number of conductors in each pole face if the machine is fitted with a compensation winding.

c) Find the number of turns in each interpole coil if the machine has interpole as well as a compensating winding.

6.12 in section 5.5 it was stated that the brushes may be shifted from the q-axis unintentionally, for example due to incorrect positioning, or they may be shifted intentionally to improve commutation, as explained in section 6.5.3. in section 5.5 it was also stated that, when the brushes are shifted, armature reaction may be magnetizing or demagnetizing. Sketch suitable developed diagrams to verify that when the brushes are shifted to improve commutation, armature reaction is always demagnetizing.

6.13 give two ways in which over commutation can occur.

6.14 on which axis of the machine does each of the following act:

(i) shunt field winding, (ii) series field winding, (iii) permanent magnets, (iv) demagnetizing armature reaction, (v) normal armature reaction, (vi) interpole winding, and (vii) compensating winding.

6.1 66. 7 ms, 1 ms. **6.2** a. 1.3m; b. (9/n) ms; c. 7.25%, 92.75%; d. 27.2 coils. **6.3** 0.5316V. **6.4** a. 1 ms; b. 24 turns. **6.5** 17 turns.

6.11 a. 59 turns; b. 66 conductors; c. 18 turns.

CHAPTER 7
POWER CONVERSION AND LOSSES

The input power to the dc machine under generator action is mechanical conversion to produce the output power; the process yields a number of losses that appear as heat which has harmful effects en the performance of the machine. In this chapter we study the conversion process and the losses associated with it.

7.1 Power balance

Most of the input power supplied to a dc machine is converted into useful output power; the remainder of the input power is lost as heat; see fig. 7.1. The principle of conservation of energy requires total power balance

$$P_{in} = P_{out} + \text{LOSSES} \qquad (7.1)$$

It is sometimes useful to think of power as 'flowing' through the machine, fig.7.2. Power flow is divided into two stages, the borderline being the actual electromechanical energy conversion process P_c is the conversion power.

$$P_c = E_A I_A = \omega_r T_d \qquad (7.2)$$

E_A, is the induced emf, T_d the developed torque, I_A the arm current, and ω_r the angular shaft speed. P_c is also called the <u>internal power</u> because it is defined within the machine; in contrast, P_{in} and P_{out} are <u>external powers</u> that can be measured. E_A and T_d in eqn.7.2 are internal quantities that cannot be measured directly. Power balance in fig. 7.2 requires that

$$P_{in} = P_c + LOSS_1 \text{ and } \qquad P_c = P_{out} + LOSS_2 \qquad (7.3)$$

The total loss is made up of 2 parts :$LOSS_1$ occurs before conversion, and $LOSS_2$ occurs after conversion. Clearly

$$P_{in} > P_c > P_{out} \qquad (7.4)$$

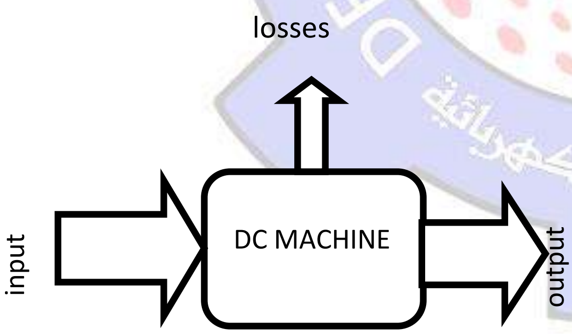

Fig. (7.1) division of power

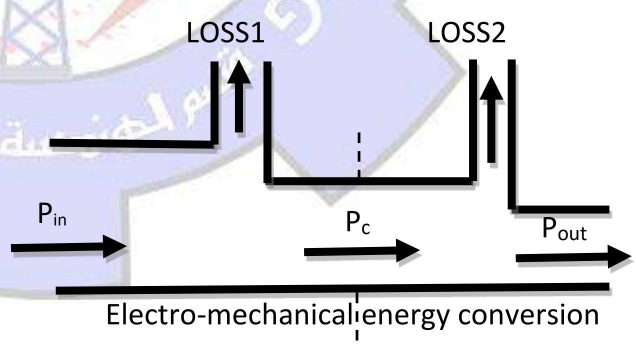

Fig. (7.2)

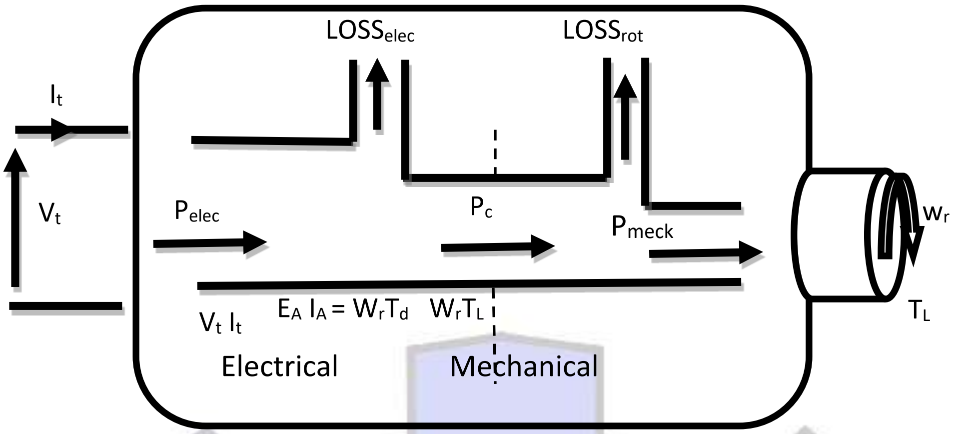

Fig. (7.3) Flow of power through dc motor

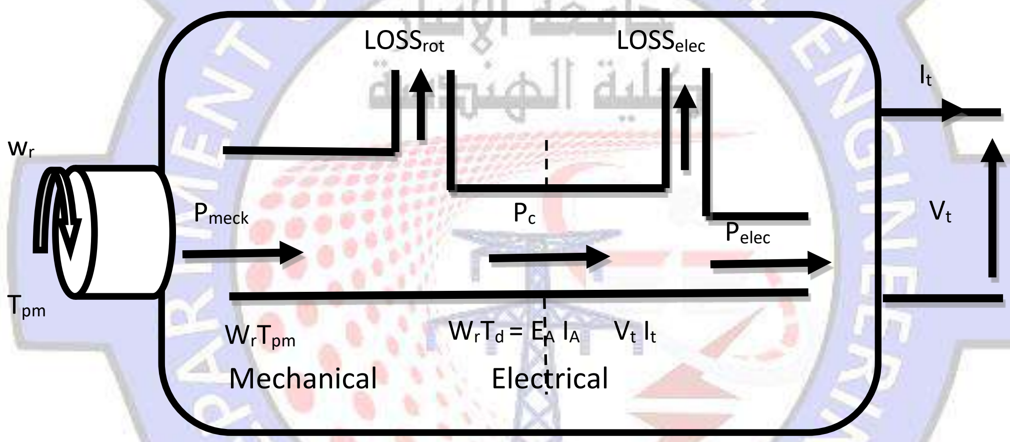

Fig. (7.4) Flow of power through dc generator

7.2 Motor Operation

The input power is electrical, and the output power is mechanical, fig. 7.3. Part of the input power is lost as electrical (copper)losses in the windings, and the remainder is available for electromechanical energy conversion; part of the converted power P_c is lost in supplying the losses due to rotation, and the remainder is available as a mechanical output power to drive the load. Note that

$$P_{mech} < P_c \rightarrow \quad \omega_r T_L < \omega_r T_d \quad \rightarrow T_L < T_d \qquad (7.5)$$

That is, the Shaft torque available at the load, T_L, is lest than developed torque T_d; the difference is needed to overcome opposing torques within the motor (such as bearing friction).

7.3 Generator Operation

The input power is mechanical, and the output power is electrical, fig. 7.4. Part of the input power is lost as rotational losses, and the remainder is available for electromechanical energy conversion; part of the converted power P_c is lost as electrical (copper) losses in the windings, and the remainder is available as electrical output power to supply the load. Note that

$$P_{mech} \geq P_c \rightarrow \omega_r T_{pm} \geq \omega_r T_d \rightarrow T_{pm} \geq T_d \quad (7.6)$$

That is, the shaft torque produced by the prime mover, T_{pm}, is greater than the developed torque T_d; the difference is the torque needed to overcome friction and other opposing torques (other than T_d).

Remark

It is good practice to keep power balance always in mind :every power we define must come from somewhere and, conversely, every bit of it must go somewhere. This is true net only in dc machines, but in all physical systems; it often helps us in understanding the processes we study.

7.4 Losses

The losses of a dc machine are of various types and occur in different parts of the machine. Although different losses are produced differently, they all appear as heat, i.e. they represent conversion to useless thermal energy. The heat generated by the losses has two major effects:

(i)Losses raise the temperature inside the machine, and thus affect the performance and life of the materials of the machine, particularly insulation. Therefore, losses determine the upper limits on machine rating.

(ii)Losses are a waste of energy, and energy costs money; therefore, losses result in a waste of money (in the operating cost of the machine).

Losses cannot be eliminated, but they can be reduced by proper design; the design must also provide for ventilation to disperse the heat generated. Thus, losses have a significant effect on the initial cost of the machine.

The cost of wasted energy in item (ii)above is important with industrial motors where the powers involved are quite high; it is not important with small control motors where the powers involved are very small. However, the temperature rise in item (i)is important for all motors.

7.4.1 Electrical Losses

Electrical losses are also called copper losses, winding losses, I^2R losses, and ohmic losses. Copper losses occur in all windings (see fig. 6.11) due to the flow of current through them; they are:

$LOSS_{arm} = I_A^2 R_A$ =armature circuit copper loss (7.7a)

$LOSS_{ser} = I_s^2 R_s$ =series field copper loss (7.7b)

$LOSS_{sh} = I_f^2 R_f$= shunt field copper loss (7.7c)

In computing $LOSS_{arm}$, R_A includes the resistances of commutating and compensating windings (if present). The series field current I_s may or may not be equal to the armature current I_A, see fig. 4.3.

The copper loss in a given wdg is proportional to the square of the current in that wdg; if the current is doubled, the copper loss increases four times. $LOSS_{arm}$ and $LOSS_{ser}$ depend on armature current, and hence they depend on the load on the machine (I_A increases with load) LOSSsh depends on the terminal voltage, and varies with its square.

The above expressions can be used to calculate copper losses using measured values of winding resistances. The wdg resistance must be at the correct wdg temperature; if the temperature at which the less is required is not known, it is assumed to be 75 ^oC. If the wdg resistance is known (say by measurement) at a temperature T_1, it can be found at a different temperature T_2 from

$$\frac{R_2}{R_1} = \frac{T_2 + 234.5}{T_1 + 234.5} \qquad (7.8)$$

The brush contact loss is also an electrical loss. Since the brush contact drop V_b is approximately constant over a wide range of armature currents, the loss is proportional to the armature current itself (and not it's square as in wdg losses):

$$LOSS_{contact} = I_A V_b \qquad (7.9)$$

7.4.2 Magnetic Losses

Magnetic losses are also called iron loses or core losses. They result from hysteresis and eddy currents in cores subjected to varying magnetization, i.e. mainly in the armature teeth and core, but also in the pole shoes (due to armature slotting -see fig. 4.2b).

Iron losses are distributed in the cores in complicated patterns, so that there are no simple formulae that give their values accurately. It is known, however, that iron losses depend on the magnetization level (flux density) in the cores, and on the frequency with which it alternates, f =np. For the hysteresis loss, we have:

$$LOSS_{hyst} \ \alpha\ f\ B^x_{MAX} \qquad (7.10)$$

Where the constant of proportionality is determined by the volume of the core and its magnetic characteristics (hysteresis loop). The Steinmetz exponent x depends on the type of iron used, and ranges from 1.5 to 2.5 (usually around 2); it is an empirical constant (obtained from experience and testing, not from electromagnetic theory). For the eddy current loss, we have:

$$LOSS_{eddy} \ \alpha\ f^2\ B^2_{MAX}\ (\text{lamination thickness})^2 \qquad (7.11)$$

Where the constant of proportionality is determined by the volume of the core and its electrical characteristics (resistivity). Clearly, thin laminations reduce eddy current losses. The armature is always laminated, and the pole shoes are usually laminated. If a motor is to be driven from a modern solid-state controlled rectifier, all cores are laminated (including poles and yoke).

7.4.3 Mechanical Losses

Mechanical losses arise from friction and windage (friction with air) during rotation. They depend on the speed of rotation, each type mechanical loss being proportional to some power of n. Bearing friction loss depends on the type of bearing used and on the viscosity of the lubricant; improper lubrication (too little or too much) increases the loss. Brush friction loss is proportional to the area of contact and to the brush pressure; it also depends on the brush and commutator materials, their slate of polisn, and the temperature at the contact surface; it is often the largest friction loss. Windage losses arise from moving the air around the armature (air friction); they depend on the shape of the rotating surface (smooth or rough).

Ventilation loss is an additional windage loss duo to fans and vent ducts used to cool the machine.

7.4.4 Stray Load Loss

Stray load losses are additional losses that occur in the machine when loaded, and cannot be included with the conventional losses listed above. They include:

-Additional core loss resulting from armature reaction distortion;

-Copper loss due to short circuit current during commutation (in commutating coils, commutator segments, and brushes);

-Non uniform current distribution in large armature conductors.

Stray load losses are small and difficult to calculate. They may be neglected for small machines, and are usually assumed 1 %of output for large machines.

7.5 Classification of Losses

Table 7.1 is a brief summary of the losses described in section 7.2; it gives their types (electrical, magnetic, or mechanics) and the main factors they depend on. The table also classifies the losses according to whether or not they are 'rotational' losses, and according to whether they are constant or vary with load. We now explain these two classifications.

7.5.1 Rotational losses

Rotational losses are the losses arising from the rotation of the armature. They include friction and windage losses, as well as core losses (in dc machines, the power lost in the core is not supplied by the source of the field, but by the torque that rotates the armature-see figures 7.3 and 7.4)

$$LOSS_{rot} = LOSS_{core} + LOSS_{mech} \tag{7.12}$$

Rotational losses increase with speed; the various component losses are functions of different powers of speed. Rotational losses exist even when the machine is running at no load because they do not depend on the armature current; therefore, they are sometimes called ‘no load losses’.

Rotational losses are supplied by the difference between the external and developed torques:

Motor (fig. 7.3) : $LOSS_{rot}=P_c-P_{mech}=\omega_r(T_d-T_L)$ (7.13)

Generator (fig. 7.4) : $LOSS_{rot}=P_{mech}-P_c=\omega_r(T_{pm}-T_d)$ (7.14)

Table 7.1 classification of losses in dc machines.

	loss	type	Rotational	With load	dependence
1	Armature circuit copper loss	Elec.	x	variable	α I2A
2	Series field copper loss	Elec.	x	variable	α I2A
3	Shunt field copper loss	Elec.	x	constant	α Vt2
4	Brush contact loss	Elec.	x	variable	α Ia
5	Hysteresis loss	Mag.	√	constant	α fBmaxx
6	Eddy current loss	Mag.	√	constant	α f2 Bmax2
7	Friction loss	Mech.	√	constant	α powers of n
8	Windage loss	Mech.	√	constant	α powers of n
9	Stray load loss		√	variable	indeterminate

Table 7.2typical values of dc machine losses for industrial meters in the range 1-100 KW. Lower percentage losses are for the higher rated motors.

Losses	% of rated power
Armature circuit electrical loss	3-6%
Shunt field electrical loss	1-3%
Rotational losses	3-13%

Table 7.3 typical efficiencies of industrial motors.

Rated power, KW	Efficiency
1	75%
50	90%
500	94%
5000	97%

7.5.2 Constant and Variable Losses

Constant losses are losses that do not change as the load on the machine changes; they are independent of armature current, and Include -mechanical losses, core losses, and shunt field loss. Variable losses are losses that increase as the load on the machine increases; they are electrical losses including armature circuit copper loss, series field loss, and brush contact loss. Copper losses increase with I_A^2, while brush contact loss increases with 1_A itself. We may therefore write:

$$LOSSES_{total} = K_0 + K_1 I_A + K_2 I_A^2 \quad (7.15)$$

The first term on the RHS represents constant losses, while the second and third terms represent variable losses. At full load, constant losses are 4-20 %and variable losses are 3-6%; see table 7.2.

Remark

Stray load losses are indeterminate functions of armature current and speed. They complicate classification, but are small enough to be neglected in most cases.

7.6 Measurement of Losses

There are a number of practical tests to measure the various machine losses. In most cases, a given test yields the sum of two or more losses together; sometimes the component losses can be separated by further testing.

In testing, it is quite easy to measure electrical Quantities (resistance, voltage, and current) and speed, somewhat difficult to measure torque, and quite difficult to measure magnetic quantities (flux and flux density). Powers are determined, on the electrical side by the product of voltage and current, and on the mechanical side by the product of torque and angular speed.

In a load test, the machine is loaded at a given speed and field excitation (i.e. field current); the input and output powers are measured. The total loss at that speed, excitation, and load is obtained from eqn. 7.1:

$$LOSSES_{total}=P_{in}-P_{out} \qquad (7.16)$$

The total loss can be separated into electrical and rotational losses by calculating I^2R products in the various windings using wdg currents measured during the load test and (hot) wdg resistances measured previously. With the electrical losses thus calculated, the rotational losses are obtained from

$$LOSS_{rot} = LOSSES_{total} - LOSS_{elec} \qquad (7.17)$$

Load tests for large machines are impractical in test labs: they require very large loads, and waste large amounts of energy. There are other tests that yield the losses individually.

In a no-load test, the machine is driven by a suitable prime mover (possibly another machine-see fig. 7.6) with Its terminals open circuited (i.e. it operates as an unloaded generator). The input power to the test machine is measured mechanically (torque and speed), or electrically by measuring the input power to the drive motor and subtracting its losses (which must therefore be known). If the test machine is unexcited then $p_{in} = LOSS_{mech}$. If the test machine is then excited but left unloaded we get $P_{in} = LOSS_{rot}$; the core loss is obtained from $LOSS_{core} = LOSS_{rot} - LOSS_{mech}$. (Exercise :suggest a test for separating the brush friction loss).

If no suitable drive (prime mover) is available, rotational losses may be obtained by running the machine as a motor with no external load; this is the running light test, or Swinburne test. The input power will mainly go to rotational losses, but there will also be a little copper loss (why?) The copper loss may be computed and subtracted from the input power to yield rotational losses. In the running light test, rotational losses cannot be separated into mechanical and core losses (why?).

As seen from the above tests, it is always possible to determine copper losses from the measured values of wag currents during the tests, and previously measured wdg resistances. Winding resistances are measured by standard methods (voltmeter-ammeter, wheatstone bridge, etc.); the wdg temperatures must be monitored at the time of resistance measurement (why?), or the measurement is made with the machine hot (for example directly after a load test).

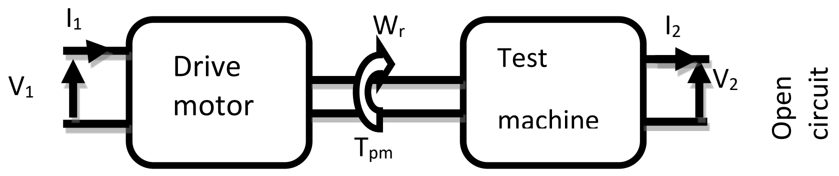

Fig. (7.5) no load test of dc machine

The no load and running light tests determine machine losses without loading it. Other tests have been devised to operate the machine at full load conditions without requiring an external load. An example of such tests is the opposition test (Kapp-Hopkinson test) which requires two identical machines. The machines are coupled mechanically, and connected in parallel with each other to the mains, fig. 7.6. By increasing the excitation for one machine and decreasing it for the other, the first will operate as a generator, and the second as a motor :the generator supplies the motor electrically, while the motor drives the generator mechanically; the power input from the mains supplies the losses to keep the system running. By suitable adjustments of the field rheostats, the armature currents can be set to their rated value. With the two machines running at or near rating (current, voltage, and speed), they will develop full-load losses, yet the disadvantages of a load test have been avoided :no external load is needed, and the mains supplies only machine losses, not their full power.

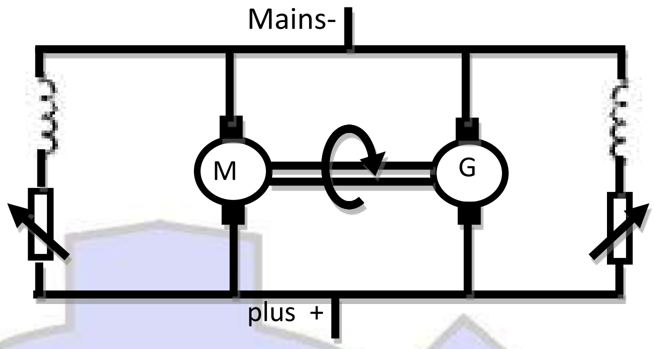

Fig. (7.6) Opposition test

In a heat run (or temperature-rise test), the machine is run at full-load to develop full-load losses; the test takes 20-60 minutes until the temperature reaches steady state corresponding to its rated operating value. The no-load and running light tests cannot replace the load test in a heat run because they do not generate all machine losses simultaneously. An opposition test, on the other hand, can be used in a heat run.

7.7 Efficiency

The efficiency of a machine is defined by

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + LOSSES} = 1 - \frac{LOSSES}{P_{in}} \qquad (7.18)$$

The first expression is general, the second is suitable for generators (P_{out} measured electrically), and the third is suitable for motors (P_{in} measured electrically). The efficiency is a fraction less than unity, and is usually expressed in percent. Table 7.3 lists typical values of dc machine efficiencies,

The overall efficiency defined, above can be analyzed into two component efficiencies corresponding to the two stages of power flow as fig. 7.2:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_c}{P_{in}} \frac{P_{out}}{P_c} \qquad (7.19)$$

For a motor, fig. 7.3, this becomes:

$$\eta = \frac{P_c}{P_{elec}} \frac{P_{mech}}{P_c} = \text{(eff. Of conversion) x (mech. Eff) .} \qquad (7.20)$$

And for a generator, fig. 7.4, it becomes:

$$\eta = \frac{P_c}{P_{mech}} \frac{p_{elec}}{P_c}$$ =(eff. of conversion) x (elec. Eff.) (7.21)

7.8 Maximum Efficiency

In section 7.4, the losses were divided into constant and variable losses (see table 7.1); the losses were then expressed as in eqn.7.15. Now consider motor operation :neglecting the small shunt field current, the input power is written:

$$P_{in} = V_t I_A \qquad (7.22)$$

Using the third expression in eqn. 7.18, the efficiency is written:

$$\eta = 1 - \frac{K_0 + K_1 I_A + K_2 I_A^2}{V_t I_A} = 1 - \frac{1}{V_t}(K_0 I_A^{-1} + K_1 + K_2 I_A) \qquad (7.23)$$

This equation gives efficiency as a function of armature current I_A; the general shape of the resulting curve is shown in fig. 7.7 (where line current is approximately equal to armature current).

To locate the point of maximum efficiency, we differentiate equ. 7.23 and equate to zero:

$$\frac{\partial \eta}{\partial I_A} = -\frac{1}{V_t}(-K_0 I_A^{-2} + K_2) = 0 \quad \Rightarrow \quad K_0 = K_2 I_A^2 \qquad (7.24)$$

Thus, maximum efficiency occurs when the copper losses $K_2 I_A^2$ equal the constant losses K_0(or, as an approximation, when variable losses equal constant losses (i.e. neglecting the brush contact loss $K_1 I_A$). Industrial machines are usually designed to have maximum efficiency for I_A between half and full load values (because the machine operates at less than full load most of the time); the exact choice is not critical because the efficiency curve is flat around maximum value, fig. 7.7.

7.9 Importance of Efficiency

For industrial motors, traction motors, and other power-application motor efficiency is quite important, but it is only one of a number of factors that determine how good a machine is; the other factors include power/weight ratio, power/cost ratio, reliability, maintenance requirements, vibration, noise, etc. For sma11 control motors, efficiency is of secondary importance; main factors of interest include accuracy, cost, size, speed of response, reliability, noise, weight, interference, etc.

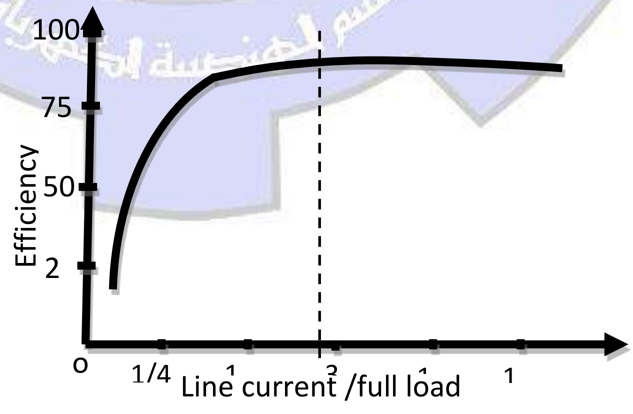

Fig. (7.7) Typical efficiency curve for a dc machine

7.10 EXERCISE:

Unless otherwise stated, assume that (a) winding resistances are given at the working temperatures, (b) the demagnetizing effect of armature reaction is negligible, and (c) the brush contact drop is 2V.

1. A 6-pole dc machine has 95 slots, the armature winding is connected in simple wave with 2 coil sides per slot per layer, and 3 turns per coil. The pole arc covers 70% of the pole pitch. The full load armature current is 600A.
   a) The armature winding is connected in simple lap; find
      1) The cross- magnetizing armature mmf at the q-axis;
      2) The cross-magnetizing armature mmf at the pole tip;
      3) The number of turns of each interpole coil;
      4) The number of conductors to be placed in each pole face to compensate for armature reaction;
      5) The number of turns of each interpole coil when there is a compensating winding (assume the compensating winding fully neutralizes the armature reaction under the pole);
      6) The cross –magnetizing and demagnetizing armature mmf's (at the q-and d-axis respectively) when the brushes are shifted 6 degrees from the q-axis to improve commutation.
   b) Repeat part (a) for simple wave connection.

<u>NB</u> list your results in a suitable table for case of comparison.

2. An armature has 268 coils and rotates at 900 rpm. Each brush covers 4 commutator segments.
   a) Find the time for one revolution.
   b) Estimate the interval during which a brush short circuits a coil.
   c) Why is your answer in part (b) approximate?
3. A 125 KW, 250V, 1800 rpm, cumulative compound generator is connected in long shunt. The armature, series field, and shunt field winding resistances are 0.025Ω, 0.01Ω, and 30Ω respectively. At rating, the shunt field current is 5 A, and the eddy current, hysteresis, frictions, and windage losses are 2KW, 1.2KW, 1.1 KW, and 700W respectively.
   a. Find the value at which the field control resistor is set.
   b. Find the electrical loss, rotational loss, and the stray load loss.
   c. Find the input horsepower, the prime mover torque, and the generator efficiency.
4. A 3/4 hp shunt dc motor runs on 60 V and draws 12 A line current; the field current is 0.4 A, and the shaft speed is 1780 rpm. The temperature of the armature winding is estimated to be 90 ^{0}c. the armature resistance is 0.32 Ω at 20 ^{0}c.
   a) Find the input power and the conversion power.
   b) Find the developed torque and the load torque.
   c) Find the conversion, mechanical, and over- all efficiency
   d) Find the current at which effecency is maximum, and find the maximum effecency. What assumption is needed for this calculation?
   e) The load on the motor is reduced so that the line current becoms 6 A; the temperature of the armature winding decreases to 70 ^{0}c. (i) find the shaft speed; (ii) repeat part (a), (b), and (c) for the new load.
5. A 20 hp shunt motor is rated at 150 V and 1400 rpm. A no-load test is performed on the macine using a 380 V drive motor to rotate it at rated speed. With the test machine unexcited, the drive motor current and losses are 2A and 260W. with the test machine field excited from a separate source, the drive motor current and losses are 5A and 400W. the armature circuit and field circuit resistances of the test motor are 0.16Ω and 50Ω respectively.
   a. Find the drive motor shaft torque in the two tests.
   b. Find the mechanical losses, core losses, and rotational losses of the test machine.
   c. Find the motor current at rating.
   d. Find the indevedual electrical losses at rating.
   e. Find the conversion, mechanical, and over all efficencies at rating.
   f. Estimate maximum efficiency. Why is your cacultion approximate ?(speed)
6. A 45 KW, 220V, 750 rpm, cumulative compound generator is connected in short shunt; the shunt field current is 10A at rating. The armature circuit and series field resistances were measured at 30 ^{0}c and

found to be 0.062Ω and 0.005Ω respectively. To measure the rotational losses, the machine was run as an unloaded motor : 240V is applied and the shunt field control resistor is adjusted for rated speed; the armature and shunt field currents were found to be 10A and 13A respectively.

a) Find the individual losses at rating.
b) Find the input power and conversion power at rating.
c) Find the conversion, electrical, and over-all efficiencies at rating.
d) Why was the running light test performed at 240 V instead of rated voltage? (emf).
e) Estimate the ratio of series field turns to shunt field turns.

Answers:

1. (a) 9.5 KAT; 6.65 KAT; 20; 22; 6;7.6 KAT, 1.9 KAT.

(b) 28.5 KAT; 19.95KAT; 59; 66; 18; 22.8 KAT, 5.7 KAT.

2. 66.7 ms; 1ms.
3. (a) 20 Ω; (b) 11.19KW, 5.00 KW, 1.25 KW; (c) 191 hp, 756N-m, 87.8%.
4. (a) 720 W, 617.9 W; (b) 3.31N-m, 3.00 N-m; (c) 85.8%, 90.5%, 77.7%. (d) 14.2A, 77.9 %; (e) (i). 1866.5 rpm; (ii). 360 W, 312.8 W; 1.60N-m, 1.30N-m; 86.9%, 81.3%, 70.7%.
5. (a) 3.41N-m, 10.23 N-m; (b) 500W, 1000W, 1500W; (c) 132A; (d) 2658 W, 258 W, 450 W; (e) 83.0%, 90.9%, 75.4%; (f) 75.8%(1432.5 rpm).

(a) 3.34 KW, 0.24KW, 0.43KW, 2.21KW , 2.37KW; (b) 53.6KW,51.2KW; (c) 95.6%, 87.9%, 84.0%; (d) (238V) ; (e) 0.0147